Hartle Solutions week 7 chapter 7

7-1. *(a) In the singular line element for the plane (7.7) show that the distance between $r' = 0$ and a point with any finite value of r' is infinite.*
(b) Find the distance between $r' = 5$ and $r' = \infty$ along the line $\phi = 0$.

Solution:

a) The singular line element is

$$dS^2 = \left(\frac{a}{r'}\right)^4 \left(dr'^2 + r'^2 d\phi^2\right) .$$

The distance along a line of constant ϕ ($d\phi = 0$) from $r' = 0$ to any finite value of r' is

$$\int dS = \int_0^{r'} \left(\frac{a}{r'}\right)^2 dr' = \infty$$

because the integral diverges at the lower limit.

b) Similarly the distance between $r' = 5$ and $r' = \infty$ along $\phi = 0$ is

$$\int dS = \int_5^{\infty} \left(\frac{a}{r'}\right)^2 dr' = a\left(-\frac{a}{r'}\right)\Bigg|_5^{\infty} = \frac{a^2}{5} .$$

Even though the coordinate range is infinite in r', the distance is finite. Another way to see this is that it's just the distance from $r = 0$ to $r = a^2/5$ in usual polar coordinates.

7-2. *The following line element corresponds to flat spacetime*

$$ds^2 = -dt^2 + 2dx\,dt + dy^2 + dz^2 \ .$$

Find a coordinate transformation which puts the line element in the usual flat space form (7.1).

Solution: Its hard to give a general prescription for solving this kind of problem. Guesswork and trial and error are the main methods. We, therefore, present some solutions without trying to explain exactly how they were arrived at.

$$ds^2 = -dt^2 + 2dx\,dt + dy^2 + dz^2$$

The transformation

$$t = t' + x' \, , \, x = x' \ , \ y = y' \ , \ z = z'$$

leads to

$$\begin{aligned} ds^2 &= -\left(dt' + dx'\right)^2 + 2\,dx'\left(dt' + dx'\right) + dy'^2 + dz'^2 \\ &= -\left(dt'\right)^2 + \left(dx'\right)^2 + \left(dy'\right)^2 + \left(dz'\right)^2 \end{aligned}$$

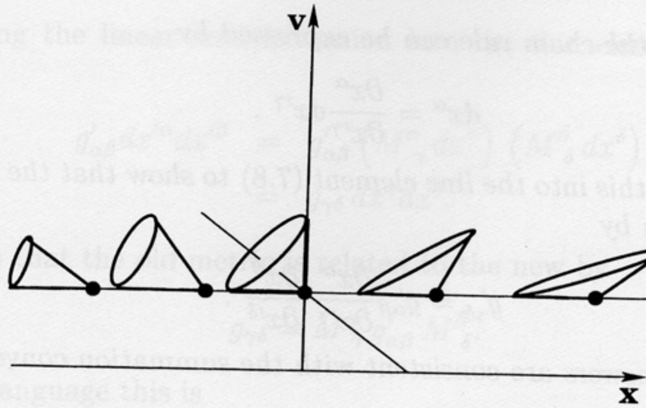
which is the standard form of the flat space line element.

Problem 7.5

Solution:

a) Light rays move on curves along which $ds^2 = 0$. These are curves with slopes

$$\frac{dv}{dx} = 0 \, , \ \frac{dv}{dx} = \frac{1}{(2x)}$$



b)

c) World lines of particles must lie inside the light cone, as the world line moving from positive to negative x shown. But there are none the other way.