Allman 1.1 Ex ,2,7,9,13;

1.1.2. a. $P_{t+1} = 2P_t$, $\Delta t = .5$ hr

b. In the following table, t is measured in half-hours.

t	0	2	4	6	8	10
P_t	1	4	16	64	256	1024
t	12	14	16	18	20	22
P_t	4096	16384	65536	262144	1048576	4194304

c. According to the model, the number of cells after ten hours is over one million. Since the observed number is around 30,000, this suggests that the model only fits well at the early stages of cell division, and that during the first ten hours (or twenty time steps) the rate of cell division has slowed. Understanding how and why this slow down occurs could be biologically interesting.

1.1.7. a. $k > 1$ and $r > 0$

b. $0 \leq k < 1$ and $-1 \leq r < 0$

c. $k = 1$ and $r = 0$

1.1.9.

t	0	1	2	3	4	5
N_t	.9613	1.442	2.163	3.2444	4.8667	7.3

1.1.13. a. The equation is precisely the statement that the amount of light penetrating to a depth of $d + 1$ meters is proportional to the amount of light penetrating to d meters.

b. $k \in (0, 1)$. The constant of proportionality k can not be greater than 1 since less light penetrates to a depth of $d + 1$ meters than to a depth of d meters. Also, k can not be negative since it does not make sense that an amount of light be negative.

c. The plot shows a rapid exponential decay.

d. The model is probably less applicable to a forest canopy, but it would depend on the makeup of the forest. Many trees have a thick covering of leaves at the tops of their trunks, but few leaves and branches closer to the bottom. This means that it is more difficult for light to penetrate near the tops of trees than it is near the bottom.

_Allman Ch 1.2, Ex 2,5,7,8,9,11

1.2.2. ΔP will be positive for any value of $P < 10$ and ΔP will be negative for any value of $P > 10$. Assuming $P > 0$ so that the model has a meaningful biological interpretation, we see that a population increases in size if it is smaller than the carrying capacity $K = 10$ of the environment, and decreases when it is larger than the environment's carrying capacity.

1.2.5. a. $\Delta P = 2P(1 - P/10)$; $\Delta P = 2P - .2P^2$; $\Delta P = .2P(10 - P)$; $P_{t+1} = 3P_t - .2P_t^2$

b. $\Delta P = 1.5P(1 - P/(7.5))$; $\Delta P = 1.5P - .2P^2$; $\Delta P = .2P(7.5 - P)$; $P_{t+1} = 2.5P_t - .2P_t^2$

1.2.6. b. The MATLAB commands `x=[0:.1:12]`, `y=x+.8*x.*(1-x/10)`, `plot(x,y)` work.

c. The cobweb diagram should fit well with the table below.

t	0	1	2	3	4	5
P_t	1	1.72	2.8593	4.4927	6.4721	8.2988

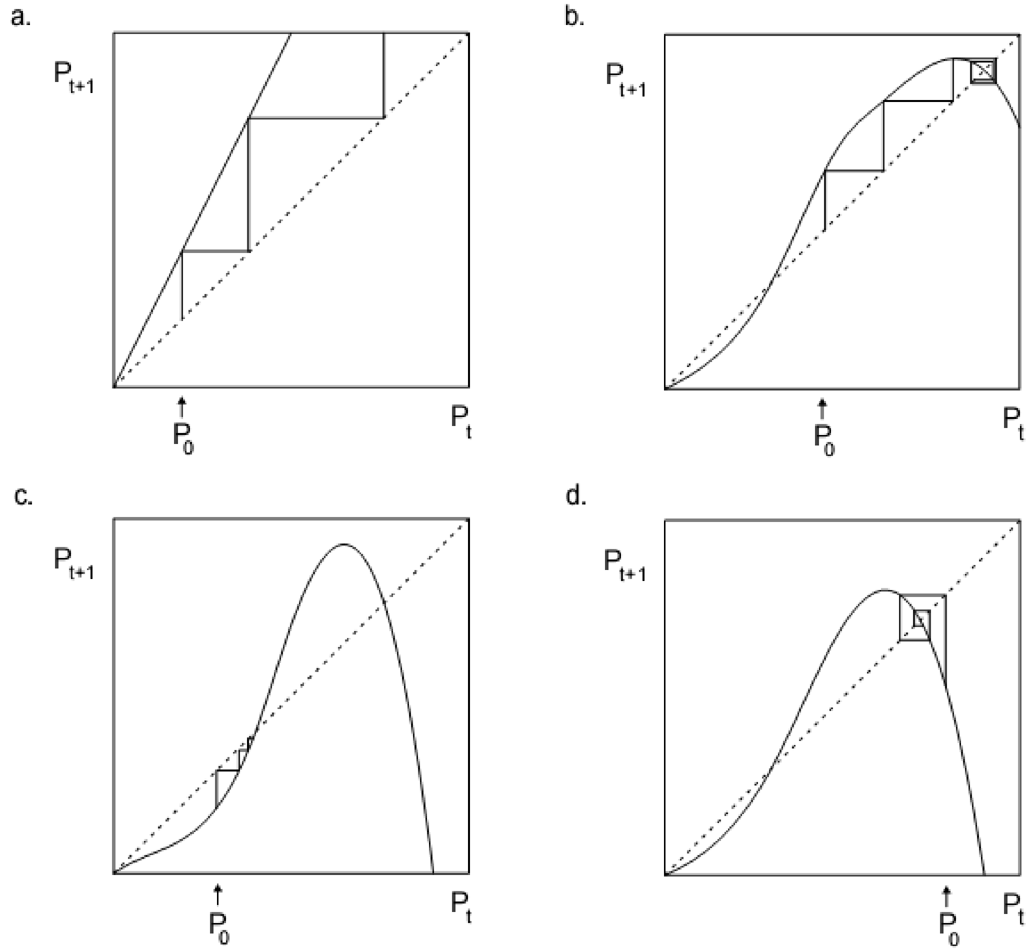
However, it is hard to cobweb very accurately by hand, so you shouldn't be too surprised if your diagram matches the table poorly. Errors tend to compound with each additional step.

1.2.7. After graphing the data, a logistic equation seems like a reasonable choice for the model. Estimating from the table and graph, $K \approx 8.5$ seems like a good choice for the carrying capacity. Since $P_2/P_1 \approx 1.567$, a reasonable choice for r is .567. However, trial and error shows that increasing the r value a bit appears to give an even better logistic fit. Here is one possible answer: $\Delta P = .63P(1 - P/8.5)$.

1.2.8. a. $M_{t+1} = M_t + .2M_t(1 - \frac{M_t}{200})$, where M_t is measured in thousands of individuals. Notice that the carrying capacity is $K = 200$ thousands, rather than 200,000 individuals. In addition, observe that if the model had been exponential, $M_{t+1} = M_t + .2M_t$, that changing the units would have no effect on the formula expressing the model.

b. $L_{t+1} = L_t + .2L_t(1 - L_t)$, where L_t is measured in units of 200,000 individuals.

1.2.9.



1.2.11. a. The equation states the change in the amount N of chemical 2 is proportional to the amount of chemical 2 present. Values of r that are reasonable are $0 \leq r \leq 1$ and $N_0 = 0$. (However, if $r = 1$, then all of chemical 1 is converted to chemical 2 in a single time step.) A graph of N_t as a function of t looks like an exponential decay curve that has been reflected about a horizontal axis, and moved upward so that it has a horizontal asymptote at $N = K$. Thus, N_t is an increasing function, but its rate of increase is slowing for all time.

b. The equation states the amount of chemical 2 created at each time step is proportional to both the amount of chemical 1 and the amount of chemical 2 present. This equation describes a discrete logistic model, and the resulting time plot of N_t shows typical logistic growth. Note that with a small time interval, r should be small, and so the model should not display oscillatory behavior as it approaches equilibrium. If N_0 equals zero, then the chemical reaction will not take place, since at least a trace amount of chemical 2 is necessary for this particular reaction. The shape of a logistic curve makes a lot

Allman 1.3, Ex. 6(a)(c),7(a),11

1.3.6. a. $P^* = 0, 15$
b. $P^* = 0, 44$
c. $P^* = 0, 20$
d. $P^* = 0, a/b$
e. $P^* = 0, (c-1)/d$

1.3.7. a. At $P^* = 0$, the linearization is $p_{t+1} \approx 1.3p_t$. Since $|1.3| > 1$, $P^* = 0$ is unstable. At $P^* = 15$, the linearization process gives

$$15 + p_{t+1} = 1.3(15 + p_t) - .02(15 + p_t)^2 \implies$$
$$15 + p_{t+1} = [1.3(15) - .02(15)^2] + 1.3(p_t) - .02(30p_t + p_t{}^2) \implies$$
$$p_{t+1} = 1.3(p_t) - .02(30p_t + p_t{}^2) \implies$$
$$p_{t+1} \approx 1.3(p_t) - .02(30p_t) = .7p_t.$$

Since $|.7| < 1$, $P^* = 15$ is stable.

1.3.11. a. Since the concentration of oxygen in the blood stream can not be more than that of the lung, B can not change by more than half the difference $(L - B)$; thus, $0 < r \leq .5$.
b. $\Delta B = r(K - 2B)$
c. If we choose an initial value $0 < B_0 < .5$ for the oxygen concentration in the bloodstream, then B steadily increases up to $B^* = .5K$. The rate of increase slows as B gets close to $.5K$. If r is increased to values just slightly smaller than .5, then B approaches equilibrium quite quickly, much more quickly than with $r = .1$.
d. $B^* = K/2$. (Note that the denominator is the total volume of the two compartments, and B^* has the correct units.) This answer makes sense in that the equilibrium concentration for B (and for L) would be (amount of oxygen)/(total volume).
e. $\Delta b = r(K - 2(K/2 + b_t)) = -2rb_t$. Equivalently, $b_{t+1} = (1 - 2r)b_t$.
f. $b_t = (1 - 2r)^t b_0$. $B_t = K/2 + (1 - 2r)^t b_0$. Note that $b_0 < 0$, since we assume that $L > B$ initially. So, since $0 \leq 1 - 2r < 1$, B increases up to its equilibrium value of $K/2$.
g. Suppose the volume of the lung is V_L and the volume of the bloodstream is V_B, then the total amount of oxygen $K = LV_L + BV_B$ and $L = (K - BV_B)/V_L$. The equation for ΔB then becomes $\Delta B = r((K - BV_B)/V_L - B)$ and the equilibrium is $B^* = K/(V_L + V_B)$, etc.