Mathematical Origins of Life Evolutionary Dynamics Test

Name:

Attempt all questions on this test. Do not leave answers blank. Credit will be given for partial answers, so show all your working.

1. Consider a simple life form on another planet whose genetic code is given by a sequence of length 10 made up of two bases A and B.

   (a) How many different genotypes are there?

   (b) What is the Hamming distance between the two genotypes

   $$ABAABAAABB \quad \text{and} \quad ABABBABBAB\,?$$

   (c) What is the dimension of the fitness landscape?

   (d) What is the highest mutation rate that is still consistent with adaptation in this fitness landscape?

2. Sea bass have the ability to produce eggs and sperm. When two sea bass mate they need to have one produce eggs and the other sperm, otherwise they will have not have a successful mating. Suppose the benefit of a successful mating is b, that the cost of producing eggs is 2 the cost of producing sperm is 1.

   (a) Suppose a bass can choose either of the two strategies: S (produce sperm) or E (produce eggs). Write down the payoff matrix for this game.

   (b) Under what conditions would either of these strategies be ESS? Would you expect such conditions to arise in nature? Why or why not?

(c) Assuming neither pure strategies are ESS, show that the mixed strategy ESS is an individual who plays S with probability

$$x = \frac{1}{2} + \frac{1}{2b}$$

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(d) Find the average fitness of players in this game. For what values of b is this fitness positive?

(e) In nature, sea bass do not play this mixed strategy ESS because it is inefficient. Explain why it is inefficient.

(f) Sea bass who pair up actually play a coordinated game. When one plays S, the other agrees to plays E. What is the average payoff for sea bass when they use this strategy?

3. Consider two types of individuals A and B with fitness $f_a = 1$ and $f_b = 1$. Let x be the relative frequency of A's and y be the relative frequency of B's.

(a) Find the equilibrium distribution of A's and B's if A mutates to B with probability 0.1 and B mutates to A with probability 0.2.

(b) How does this equilibrium shift if B has fitness $f_B = 1.2$?

4. Consider the prisoner's dilemma game represented by the situation when a player can either, cooperate by offering a benefit b to another player at cost c to itself, or defect by offering nothing and incurring no cost. As usual we assume $b > c$.

   (a) Under what conditions is Pavlov an ESS against All-D in this game?

   (b) Find the sequence of moves when the following two deterministic strategies play each other for 6 games: Anti-Pavlov: $(0, 1, 1, 0)$ and Anti-Tit4Tat: $(0, 1, 0, 1)$. Assume both strategies start by defecting.

   (c) Find the payoff matrix and determine the conditions for which either of them is an ESS.

Mathematical Origins of Life Evolutionary Dynamics Take Home Test

Name:

- Attempt both questions on this test. Do not leave answers blank. Credit will be given for partial answers, so show all your working and explain your reasoning.
- This is an open book test. You may consult your textbooks, notes, homework and workshop solutions, but you should not consult other people.
- Your completed test is due at 9:00 am on Wednesday May 12th.

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1. An evolutionary game is played with 3 strategies A, B and C occurring with relative frequencies x, y and z respectively. The payoff matrix is given below.

	A	B	C
A	-2	4	3
B	0	2	4
C	-2	0	1

   (a) Consider each of the three possible two strategy games corresponding to the case when one of these strategies is extinct. What are the pure or mixed ESS's when A plays B, when A plays against C, and when B plays against C?

   (b) As a three person game are there any pure ESS's? Show that there is no equilibrium of the three strategies when they are all equally fit?

   (c) Draw the phase simplex for this game.

2. Given the two stochastic reactive strategies A: $S(1, \frac{1}{2})$ and B: $S(0, \frac{1}{2})$ playing the repeated prisoners dilemma game with payoff matrix

	C	D
C	3	0
D	5	1

   (a) Find the equilibrium state vector $\vec{x}^*$ when A plays against B.

   (b) Find the game matrix for the expected payoffs in this game.

   (c) Are either strategy a pure ESS? If not find the mixed ESS.