1. Consider an interaction game where an individual first assesses the size of the opponent before deciding to escalate the interaction to a fight, or withdraw. The winner gains the resource worth 2 fitness points, and the loser pays a cost 6 fitness points. Suppose the larger animal wins with probability p, with $p > 0.5$.

   (a) Consider two strategies "Always escalate" (ie Hawk), and "escalate if the opponent is smaller, withdraw if the opponent is larger" (ie Conditional Hawk). Write down the game matrix for this interaction assuming that 50% of the time an opponent is larger and 50% of the time an opponent is smaller). (Hint: When a Conditional Hawk Meets a Hawk half the time the Hawk is larger and the Conditional Hawk leaves without fighting, half the time the opponent is smaller so the fight happens. When the fight happens the larger animal wins $b = 2$ with probability p and loses $c = 6$ with probability $(1 - p)$. Use these observations to calculate the average payoff for the Conditional Hawk in this interaction, and then repeat for all possible interactions).
   Hawks fight regardless so the payoff for hawks is

$$E(H, H) = \tfrac{1}{2}\,b - \tfrac{1}{2}\,c = 1 - 3 = -2$$

   . When a Conditional Hawks meets a Hawk, it will only stay to fight if it is larger, which happens half the time. It then wins with probability p and loses with probability $1 - p$. Thus

$$E(CH, H) = \tfrac{1}{2}\,(bp - (1 - p)c) = \tfrac{1}{2}\,(2p - 6 + 6p) = 4p - 3$$

   . Similarly, when the Conditional Hawk stay the Hawk will win with probability $1 - p$ and lose with p (since it is smaller) and when the Conditional Hawks leaves the Hawk will get the entire benefit. Thus

$$E(H, CH) = \tfrac{1}{2}\,(b(1 - p) - pc) + halfb = \tfrac{1}{2}\,(2 - 2p - 6p) + 1 = 2 - 4p$$

   Finally, two Conditional Hawks never fight – the smaller always withdraws. This happens half the time so

$$E(CH, CH) = \tfrac{1}{2}\,b = 1$$

   Thus the payoff Matrix is:

	H	CH
H	-2	$2 - 4p$
CH	$4p - 3$	1

   (b) Under what conditions on p, if any, is it possible for Hawk to be an ESS? Hawk is ESS if $-2 > 4p - 3$ or $4p < 1$ or $p < \frac{1}{4}$. This is never true since $p > \frac{1}{2}$.

   (c) Under what conditions on p, if any, is it possible for Conditional Hawk to be an ESS? CH is ESS if $1 > 2 - 4p$, which means $4p > 1$ or $p > \frac{1}{4}$. This is always true since $p > \frac{1}{2}$.

(d) Consider the additional strategy "always withdraw if opponent escalates" (ie Dove). Write down the payoff matrix. For which values of p do Doves do better against Hawks than Conditional Hawks?

When a Conditional Hawk meets a dove, half the time it attacks and the Dove withdraws. The other time it withdraws and the Dove keeps the reward. Thus the payoff to both is $\frac{1}{2}b = 1$. The payoff matrix is:

	H	CH	D
H	-2	$2-4p$	2
CH	$4p-3$	1	1
D	0	1	1

Doves to better against Hawks than Conditional Hawks do if $0 > 4p - 3$, or $p < \frac{3}{4}$.

(e) For the case where there are no Hawks, Doves and Conditional Hawks are equally fit. However, if the proportion of Doves is high enough, it is possible for Hawks to invade a mixed population of Doves and Conditional Hawks. Let x be the proportion of Doves and $1-x$ be the proportion of Conditional Hawks. For what value of x would a mutant Hawk be fitter than the Doves and Conditional Hawks? Express your answer in terms of p, and find the value of x for the cases where $p = 0.5$ (size doesn't matter) and $p = 1$ (size is everything).

The fitness of a Hawk, f_H will be:

$$f_H = (2-4p)(1-x) + 2x = 4p\,x + 2 - 4p$$

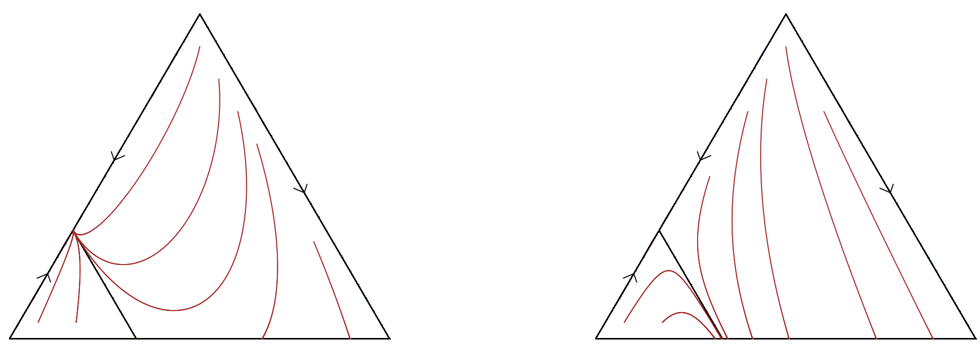
The fitness of Doves in this population is $f_D = 1$. So we want

$$f_H > f_D \Rightarrow 4p\,x + 2 - 4p > 1 \Rightarrow x > \frac{4p-1}{4p}$$

If $p = 0.5$ this would mean $x > 0.5$. If $p = 1$ then $x > \frac{3}{4}$.

(f) Sketch the dynamics of this three strategy system on a phase simplex.

The phase simplex will look a little different depending on whether doves or conditional hawks do better against hawks. The plot on the left below shows the dynamics for $p = 0.5$ when doves do better and the one on the right shows the dynamics for $p = 1$ when conditional hawks do better. In each diagram, the vertices are Hawk, Conditional Hawk and Dove, starting from the top an moving clockwise.



(g) There is a fourth strategy: ”escalate if the opponent is larger and withdraw if the opponent is smaller”. What name might you give this strategy. Write down the full 4×4 matrix for interactions involving all these strategies. Can this new strategy be an ESS when paired with any of the other strategies?

Lets call this strategy Fool. The payoffs will be similar to CH, but with p replaced with $1 - p$. The payoff matrix is:

	H	CH	D	F
H	-2	$2 - 4p$	2	$4p - 2$
CH	$4p - 3$	1	1	1
D	0	1	1	1
F	$1 - 4p$	1	1	1

Fool is an ESS if $4p - 2 < 1$, or $p < \frac{3}{4}$