## 1. Warmup

For each of the following games identify any dominated strategies, write down the reduced matrix and find all saddle points.
(a)

|  | A | B | C | D |
| :---: | :---: | :---: | :---: | :---: |
| A | 5 | 2 | 2 | 4 |
| B | 6 | 2 | 4 | 3 |
| C | -3 | 1 | 7 | 5 |
| D | 7 | 0 | -1 | 0 |

(b)

|  | A | B | C | D |
| :---: | :---: | :---: | :---: | :---: |
| A | 3 | -4 | -10 | -8 |
| B | -6 | -1 | -2 | -5 |
| C | 0 | -13 | 1 | -12 |
| D | 2 | -2 | 0 | -4 |

(c)

|  | A | B | C | D |
| :---: | :---: | :---: | :---: | :---: |
| A | 3 | -2 | -3 | 5 |
| B | 1 | -4 | -2 | 6 |
| C | 2 | 0 | 1 | -1 |

## 2. Stripped Down Poker

The following game of poker illustrates how bluffing games can be analyzed using game theory. Stripped down poker is played between two players with a deck that consists of 4 kings and 4 twos. To start, each player puts $\$ 1$ in the pot (the ante). Then Rose is dealt a single card and after she looks at it she can choose to fold or bet. If she folds Colin gets the pot and she loses her dollar (and he gains it!). If she chooses to bet she places an additional dollar in the pot. Now Colin can choose to fold or call. If he folds Rose gets the pot and wins the $\$ 1$ that Colin initially placed in the pot. If Colin chooses to call he must place an additional $\$ 1$ in the pot. Now Rose's card is revealed. If it is a King she keeps the pot and gains a net $\$ 2$ if it is a two she loses the pot and loses a net $\$ 2$.
(a) Find a partner and place this game 10 times as Rose and 10 times as Colin. Record your winnings. Also, Rose should record how many times she bet with a king and how many times she bet with a two. Colin should record how many times he called and how many times he folded.
(b) To analyze this game recognize that Colin only ever has two strategies: fold and call. Because Rose may do something different depending on whether she sees a king or a two, she actually has four strategies. She can bet regardless of the card she has (All-B), she can fold regardless of which card she has (All-F), she can bet with a king but fold with a two (BKFT), and finally she can fold with a king, but bet with a two (FKBT). Find the expected average payoffs for Rose for each of the outcomes (there are eight), assuming she gets a king half the time and a two half the time.
As an example, the payoff for the outcome corresponding to Rose choosing BKFT and Colin calling can be found as follows: When she gets a king she will bet, and since Colin calls, he will lose and Rose will get $\$ 2$. However, when she gets a two she will fold when she uses this strategy and hence, she will lose a dollar. So on average she will win $\frac{1}{2}(\$ 2)+\frac{1}{2}(-\$ 1)=\$ 0.5$. For this combination of strategies her payoff is $\$ 0.5$. Calculate the payoffs for the other outcomes and put them into a 4 x 2 payoff matrix.
(c) Once you have your game matrix, eliminate any dominated strategies.
(d) In the reduced payoff matrix, are there any equilibria?
(e) If Colin chooses to play a mixed strategy where he folds half the time and calls half the time, what will be Rose's expected payoff be for each of her strategy choices.

