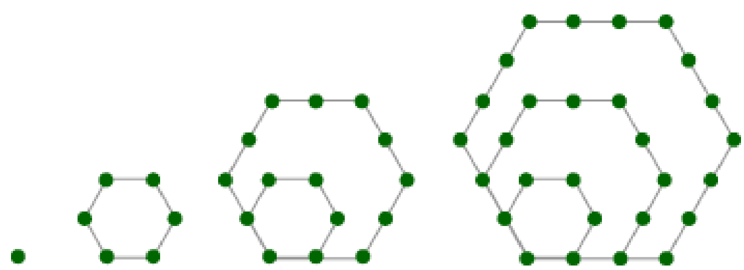
Please answer the following questions by Tuesday October 13th. Show all your work on a separate piece of paper.

1. In 1990 the diameter of the trunk of an old growth cedar tree was measured to be 49 cm. In 2002 its diameter was 55 cm. Assuming linear growth find an expression for the diameter of the tree in terms of the number of years, n since 1990. When will the tree be 100 cm wide?
   In 12 years the diameter grows by 6 cm. Assuming linear growth this means the diameter increases by 0.5 cm per year. The initial diameter (in 1990) is 49 cm and the common difference (or rate) is 0.5 so the sequence for the diameter after n years is
   $$D_n = 49 + 0.5n$$
   Note in this problem the first term in the sequence (in 1990) corresponds to $n = 0$ and not $n = 1$, this is why we add $0.5n$ and not $0.5(n-1)$.
   The tree will reach 100 cm when $D_n = 100 \Rightarrow 49 + 0.5n = 100 \Rightarrow n = 102$. The tree has will have a diameter of 100 cm after 102 years (in 2092).

2. Identify each of the following as geometric or arithmetic, write down a general formula for the nth term u_n and the nth partial sum S_n
   (a) $14 + 25 + 36 + \cdots$
   This is an arithmetic sequence, with common difference $d = 11$ and first term $a = 14$. Thus $u_n = 14 + (n-1)11 = 3 + 11n$ and $S_n = n(14 + (3 + 11n))/2 = n(17 + 11n)/2$
   (b) $27 + 36 + 48 + \cdots$
   This is a geometric sequence with common ratio $r = 4/3$ and first term $a = 27$. Thus the nth term is $u_n = 27(4/3)^{n-1}$ and the partial sum is $S_n = 27((4/3)^n - 1)/((4/3) - 1) = 81((4/3)^n - 1)$.

3. Find the limit of the following infinite series
   $$9 + 6 + 4 + \cdots$$
   This is an infinite geometric series with first term $a = 9$ and common ratio $r = \frac{2}{3}$. Since $r < 1$ this series converges. The limit is
   $$S_\infty = \frac{9}{1 - \frac{2}{3}} = 27$$

4. The following figures illustrate what are called hexagonal numbers. Find a general expression for the nth hexagonal number. Hint: Use the fact that the hexagonal numbers are the sum of an arithmetic progression.



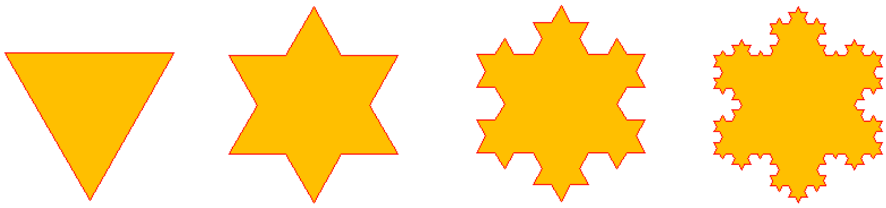
Thinking about how much is added to get from one figure to the next we see the nth hexagonal number is

$$s_n = 1 + 5 + 9 + 13 + \cdots$$

This is an arithmetic series with first term $a = 1$ and common difference $d = 4$ so

$$s_n = \frac{n(2 + 4(n-1))}{2} = n(2n-1)$$

5. The following series of figures illustrates how to construct what is called the Koch snowflake. In the second generation three triangles, each with an area 1/9th the original triangle, are added to the three sides. In the third and subsequent generations smaller triangles, which are 1/9th the area of the previous triangles are added to each edge. Find the area added each generation and then and hence find an expression for the total area if this process is carried on indefinitely. Hint: The areas added are in a geometric progression. Find the first term and the common ratio and then evaluate the infinite sum. This is may be challenging problem, so play around with it for a while. Redrawing the figures and counting the new triangles at each step will help. Give it your best shot!



Consider how much area is added in moving from one figure to the next. Assume the initial triangle has area 1. In the second figure we add three triangles, each with area $\frac{1}{9}$, so the area of the second figure is $1 + \frac{3}{9}$. In the third figure we add 12 triangles (4 for each of the triangles added the last time), each of area $(\frac{1}{9})^2$. So the total area is $1 + \frac{3}{9} + 12(\frac{1}{9})^2$. For the fourth triangle it's hard to see, but following that pattern we add 4 triangles of area $(\frac{1}{9})^3$ for each of the triangles that were added last time. That is we add $4 \times 12 = 48$ little triangles, so that the total area is

$$1 + \frac{3}{9} + 12(\frac{1}{9})^2 + 48(\frac{1}{9})^3$$

Everything after the 1 is an infinite geometric series with $a = \frac{3}{9}$ and $r = \frac{4}{9}$ so that $s_\infty = \frac{\frac{3}{9}}{1-\frac{4}{9}} = \frac{3}{5}$. Therefore the total area is $1 + 3/5 = \frac{8}{5} = 1.6$.