

1.



A conch shell has an opening that is 3 in long, and weighs 4 lbs. A geometrically similar shell has an opening that is 5 cm. What do you estimate its weight to be?

The linear scale factor is $5/3$, and since weight is proportional to volume which scales with the cube of the linear dimension, the new weight will be $4(5/3)^3 = 500/27 = 18.5$ lbs.

2. A coffee company makes three sizes of coffee. Small is 8 oz, medium is 12 oz and large is 16 oz. If they are placed in geometrically similar cups, and the small is 4 inches tall, how tall will the medium and large be?

Since the cups are geometrically similar, volume is proportional to the cube of the height. Thus $V \propto h^3$ and hence $h \propto V^{1/3}$. Now the volume scaling factor for the medium compared to the small is $12/8=3/2$. Thus the height scale factor will be $(3/2)^{1/3} = 1.145$. Thus the medium cup will be $4(1.145)=4.6$ inches tall. The volume scaling factor for the large compared to the small is $16/8=2$. Thus the large cup will be $4(2)^{1/3} = 5$ inches. It may seem remarkable that a 4 inch cup holds 8 oz and a 5 inch one holds 16. But that's how scaling works!.

3. While Galileo demonstrated that in the absence of air resistance falling objects should have the same acceleration regardless of their size or mass. This does not fit with our observations because air resistance is often important. Given enough time, all objects eventually reach terminal velocity. The terminal velocity of a falling object is directly proportional to the square root of its mass and inversely proportional to the square root of its surface area.

- (a) Write down an expression showing how terminal velocity an object depends on mass and surface area.

$$v_t \propto \sqrt{\frac{m}{S}} \text{ or } v_t = k\sqrt{\frac{m}{S}}, \text{ where } k \text{ is a proportionality constant.}$$

- (b) Two balloons have identical mass but one has twice the radius of the other. If the smaller balloon falls at 1.0 m/s, how fast does the larger balloon fall?

$$\text{Since } v_t \propto \frac{1}{\sqrt{S}} \text{ and } S \propto r^2, \text{ then } v_t \propto \frac{1}{\sqrt{r^2}} = \frac{1}{r}. \text{ Since } r \text{ is doubled, then } v_t \text{ is halved.} \\ v_t = 1.0/2 = 0.5 \text{ m/s.}$$

- (c) Two balls have identical radius, but one is twice the mass of the other. If the lighter ball falls at 4 m/s, how fast does the heavier ball fall?

$$\text{Since } v_t \propto \sqrt{m} \text{ and } m \text{ is doubled, then } v_t \text{ is larger by a factor of } \sqrt{2} = 1.41. \text{ So} \\ v_t = 4(1.41) = 5.7 \text{ m/s.}$$

- (d) In a light rain, small raindrops are 0.5 mm and fall at 3 m/s. Large raindrops are 2.0 mm. How much heavier are larger rain drops? How much more area do they have? With what speed does the larger drop fall?.

$$\text{Since the linear scale factor is } 2/0.5 = 4, \text{ and weight is proportional to the cube of the radius, the weight will be larger by a factor of } 4^3 = 64. \text{ Similarly area, which is proportional to the square of the radius, will be } 4^2 = 16 \text{ times larger. So it follows that the speed will change by factor } \sqrt{64/16} = 2. \text{ The larger drop falls twice as fast or } 6 \text{ m/s.}$$

4. A spherical raindrop evaporates at a rate which is directly proportional to its surface area. The evaporation rate is 1.0 mg/min for a raindrop with radius 2.0 mm.

- (a) Write down expression for the evaporation rate E as a function of the radius of the drop r in the form $E = kr^p$. Using the values given, find the value of the proportionality constant k .

Since $E \propto S$ and $S \propto r^2$ then $E \propto r^2$ so $E = kr^2$.

To solve for k substitute $r = 2.0$ mm and $E = 1.0$ mg/min.

$$1.0 \text{ mg/min} = k(2.0 \text{ mm})^2 \Rightarrow k = \frac{1.0 \text{ mg/min}}{4.0 \text{ mm}^2} = 0.25 \text{ mg/min/mm}^2$$

- (b) If the radius of the raindrop were double the size, by what factor would the evaporation rate change?

Since evaporation rate is proportional to the radius squared then doubling the radius will quadruple the evaporation rate.

- (c) If the volume of the raindrop were double the size, by what factor would the evaporation rate change?

Since $E \propto r^2$ and $V \propto r^3$. Then $r \propto V^{1/3}$. Thus $E \propto V^{2/3}$ Then if V is increased by scale factor 2 it follows that E is increased by scale factor $2^{2/3} = 1.36$ so the evaporation rate is increased by 36%.