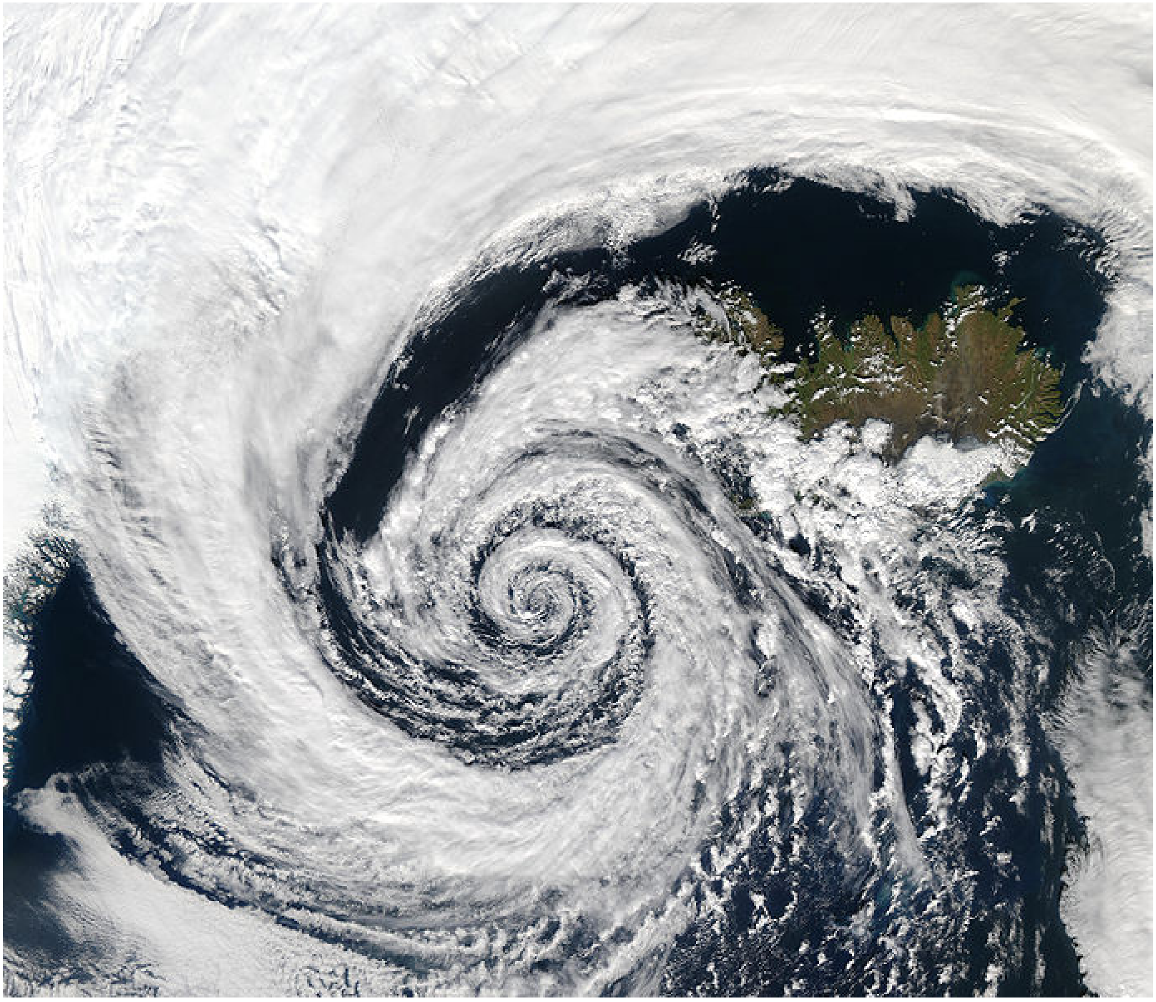
Answer the following questions on a separate sheet of paper and turn in your responses by 9:00 am on Wednesday February 9th.

1. A measuring tape with a thickness of 1.5 mm, is coiled around a rod with radius 2.0 cm. Find an expression for the radius of the roll of tape as a function of the number coils n. What type of spiral is this? What is the radius if there are 20 turns?
   The radius grows by 1.5 mm (or 0.15 cm) each turn, starting from 2.0 cm. This is an Archimdean spiral with equation $r = 2.0 + 0.15n$. After 20 turns $r = 2 + 0.15(20) = 5$ cm

2. A snail shell grows in a spiral by accretion. If the radius of the shell increases by 10% (ie a factor of 1.1) every 30^0 Find an expression for its radius as a function of angle θ, measured from a starting radius of 2 cm. What type of spiral is this? How large is the shell if it makes two complete turns beyond radius 2 cm?
   Since the shell grows by a constant factor it is exponential growth. $r = 2(1.1)^{\theta/30}$. This is a logarithmic spiral. Two turns is 720 degrees, so $r = 2(1.1)^{720/30} = 2(1.1)^{24} = 19.7$ cm.

3. For the following picture of a hurricane, draw a smooth curve roughly around the outer edge of the spiral cloud staring from the middle of the left hand side of the picture until you reach the center. Now measure the radius of the spiral every 90^o. Find the ratio between neighboring terms? Is this a reasonably good logarithmic spiral? If so, find its equation.



4. Answer the following questions based on the parastichies patterns you generated with your group in this week's workshop.

   (a) Which divergence angles result in parastichies that are straight lines? How many lines are there in each case. Explain, mathematically, why they line up in a straight line. Explain how the divergence angle relates to the number of straight lines that emerge.
   The divergence angles with which divide evenly into 360 give straight line parastichies (45, 90 and 120) because then after completing 360 degrees they repeat. 45 degrees goes into 360 8 times, so their are 8 parastichies. Similarly 90 degrees give 4 parastichies and 120 gives 3 parastichies. 135 and 150 degrees also give straight line parastichies: 135/360=5/8 so it forms 8 parastichies and 150/360=5/12, so it forms 12 parastichies.

   (b) When the divergence angles are changed by 5^o describe the changes in the parastichies pattern you observe. Is the shape of the parastichies the same? In which cases do the number of parastichies stay the same and in which cases do the number of parastichies change? Explain mathematically.
   The parastichies are now spirals. Since $50/360 = 5/36 \approx 1/7$ there are 7 parastichies. Similarly $\frac{95}{360=19/72\approx 1/4}$, so there are 4 parastichies. $125/360 = 25/72 \approx 1/3$, so there are 3 parastichies,

$$\frac{140}{360} = \frac{7}{18} = \frac{1}{\frac{18}{7}} = \frac{1}{2+\frac{4}{7}} = \frac{1}{2+\frac{1}{\frac{7}{4}}} = \frac{1}{2+\frac{1}{1+\frac{3}{4}}} \approx \frac{1}{2+\frac{1}{2}} = \frac{2}{5}$$

   So there are 5 parastichies. Finally $155/360 = 31/2 \approx 3/7$, so there are 7 parastichies.

   (c) In the case where the divergence angle is 137.5^o you should be able to see two different ways to connect the dots to form parastichies. How many spirals do you get by each method of connecting the dots? Are these numbers familiar?
   You should see 5 or 8 or 13 parastichies depending on how you draw it. These are Fibonacci numbers.

   (d) Of all the patterns which results in the arrangement that is most uniformly packed?
   The 137.5 degree divergence leads to the most uniformly packed arrangement.

5. For the divergences angle of 137.5^o the number of spirals in each way of forming the parastichies should be neighbouring Fibonacci numbers. There is a mathematical reason for this

   (a) For any given divergence angle α there is a complementary angle $360^o - \alpha$ which would generate an identical pattern. For example a divergence angle of 270^o generates the same pattern as a divergence angle of 90^o. Why?
   Because going clockwise by αwill give the same spiral as counter clockwise by $-\alpha$ (except spiraling the opposite way).

   (b) For the divergence of angle of 137.5^o find the complementary angle. Find the ratio of the complementary angle to the divergence angle in this case. What is this number? What is its connection to Fibonacci numbers?
   The complementary angle is $360 - 137.5 = 222.5$ The ratio is $222.5/137.5 \approx 1.618$ which is close to the golden ratio. The ratio of Fibonacci numbers is close to the Golden ratio also.

6. Create a drawing on a blank piece of paper of the pattern that emerges for the divergence angle 137.5^o. Replace the dots in your diagram with some representation of a plant structure

of your choice (eg seeds, scale, petals, or florets). Consider altering the size, shape and color of the structures as you move radially outward to make your pattern more natural. Be creative.