For this assignment you will investigate the properties of a variety of models for growth by using Excel to generate plots. It may help to complete the introduction to graphing with Excel tutorial before doing this assignment.

Answer each question in a separate sheet (but the same Excel file). Briefly describe your observations in a text box in Excel. Name your file using the naming convention

`Graphing_Growth_LastName_FirstName.xls`

Upload it to our class Moodle site by Tuesday, January 18th.

1. Each of the following recursive formulas are of the form $P_n = P_{n-1} + d$ and describe models of linear growth or decay of some quantity over time. Generate the first 100 terms of the following models and plot P_n vs n on the same graph. In a text box describe the main features of these models of growth. Describe how changing the initial value P_0 and the constant increase/decrease (or growth rate) d affect the nature of the graphs. Try different values to test your claims. What limitations do these models have for representing real growth.

   (a) $P_n = P_{n-1} + 5,\ P_0 = 0$

   (b) $P_n = P_{n-1} + 2,\ P_0 = 50$

   (c) $P_n = P_{n-1} - 2,\ P_0 = 100$

2. Each of the following recursive formulas are of the form $P_n = rP_{n-1}$ and describe models of exponential growth or decay of some quantity over time. Generate the first 100 terms of the following models and plot P_n vs n on the different graphs. In a text box describe the main features of these models of growth. Describe how changing the initial value P_0 and the growth factor r affect the nature of the graphs. Try different values to test your claims. What limitations do these models have for representing real growth or decay for long periods of time.

   (a) $P_n = 1.05P_{n-1},\ P_0 = 10$

   (b) $P_n = 0.95P_{n-1},\ P_0 = 100$

   (c) $P_n = -0.95P_{n-1},\ P_0 = 50$

3. Now let's consider a model of the form, $P_n = rP_{n-1} + d$, which combines linear growth with exponential growth or decay. Generate the first 100 terms of the following models and plot P_n vs n on different graphs. In a text box describe the main features of these models of growth. Describe how changing the initial value P_0 and the growth factor r affect the nature of the graphs. Which graphs show the value of P_n converging to a limit? Can you predict the limit based on the parameters? Describe the way in which the values converge to a limit. Explain why you think that is.

   (a) $P_n = 1.05P_{n-1} + 20,\ P_0 = 10$

   (b) $P_n = 0.95P_{n-1} + 20,\ P_0 = 100$

   (c) $P_n = -0.95P_{n-1} + 20,\ P_0 = 50$

4. Many biological populations, and growth processes, exhibit exponential growth when the population or object is relatively small. However, this kind of growth cannot be sustained indefinitely due to limited resources. A more realistic model is given by modifying the exponential growth model by adding a term: $P_n = rP_{n-1} - aP_{n-1}^2$. The second term in this formula represents the limitation to growth that comes about due to competition for scarce resources. The parameter a represents the chances of their being competition for resources. Generate the first 100 terms of the following models and plot P_n vs n on different graphs. In a text box describe the main features of these models of growth. Describe how changing the initial value of the growth factor r affect the nature of the graphs. Which graphs show the value of P_n converging to a limit? Describe the way in which the values converge to a limit or limits if it does. Investigate other parameter values.

   (a) $P_n = 1.05P_{n-1} - 0.001P_{n-1}^2,\ P_0 = 10$

   (b) $P_n = 3.2P_{n-1} - 0.001P_{n-1}^2,\ P_0 = 10$

   (c) $P_n = 3.5P_{n-1} - 0.001P_{n-1}^2,\ P_0 = 10$

5. The previous model is an example of how growth can be limited due to scarcity of resources. There is another way that growth can be limited, and this is from the interaction with another species or growing object. Consider the population P_n to be prey for some other species of predators R_n. Predators eat prey and so reduce their ability to grow to their natural limit. Predators are sustained by prey, but would die out in the absence of prey. The following model captures these features:

$$
\begin{aligned}
P_n &= 1.05P_{n-1} - 0.001P_{n-1}^2 - 0.001P_{n-1}R_{n-1} \\
R_n &= 0.95R_{n-1} + 0.003P_{n-1}R_{n-1}
\end{aligned}
$$

   Look at the parameter choices and make sure you understand why this model captures the features that are indicated. Now, generate the first 500 terms of P_n and R_n, using the initial values $P_0 = 10$ and $Q_0 = 10$ (Make one column for R_n and one for P_n). Make a plot of P_n vs n and R_n vs n on the same graph, and describe your observation. It is quite common to create what is called a phase plot to describe the growth of interacting population. A phase plot shows how one dynamical variable varies as another one varies. Plot R_n on the y-axis and P_n on the x-axis. Describe the phase plot for this model. What is happening to the system? Change the parameter values to see how the behavior of the system changes.