Introduction:

The simple pendulum, with its regular clockwork motion is the epitome of deterministic dynamical systems. Newton's laws allow us to determine its period and predict its subsequent motion. Small variations in initial conditions result in only small variations in the motion. In this lab we will investigate how the motion of a simple pendulum depends on its mass, the amplitude of oscillations and the length of the pendulum. We will then explore how the introduction of non-linear forces into this system can lead to chaotic and unpredictable motion. In particular we will investigate under which conditions the pendulum system becomes sensitive to initial conditions.

Procedure:

Simple Pendulum

1. First, make a hypothesis about how you think the period of a pendulum depends on each of the following variables: the mass of the pendulum, the length of the string and the amplitude of the oscillations. Make your hypothesis quantitative.
2. Set up a simple pendulum with a paperclip or washer on the end of a thread. The thread should be about a metre long. Record the length, measuring from the centre of the paperclip or washer to the top of the string.
3. Release the pendulum from rest at an angle of about 30 degrees from the vertical, and describe its subsequent motion.
   a) Does it return to its initial position?
   b) What happens to the motion over time?
3. Now repeat. Measure the time for 10 oscillations of the pendulum for four different initial amplitudes, between about 10 and 70 degrees. Does period depend on amplitude? If so, how?
4. Now attach a second paperclip or washer at the same location you have attached the first, without changing the length. Release the pendulum from about 30 degrees and time 10 oscillations. Repeat the procedure with three paperclips/washers. Does period depend on mass? If so how?
5. Using one paper clip repeat this procedure of releasing the pendulum from about 30 degrees for a total of 5 different lengths, between 1.0 m and 0.2 m. How does the period vary with length? Find a proportional expression relating length to period, of the form $T^p \propto L$, where T is the period and L is the length of the pendulum and p is a whole number (-3,-2,-1,0,1,2,3 etc) that you should find by trial and error, based on your hypothesis or observations. Recall, if two quantities are proportional, then increasing one by a certain factor increases the other by the same factor – so that $T^p / L =$ constant.

Chaotic Pendulum

1. Set up the pendulum again so that the length is about 50 cm and with one paperclip or washer. Make sure the paperclip/washer is suspended about 2-3 cm above the table surface or floor.

2. Mark off a piece of paper in square inch sections and tape it to the table surface so that the centre of the paper is directly below the paperclip/washer.

3. Place two magnets 8-10 cm apart spaced equidistant from the centre of the paper aligned along the long axis of the paper. Careful, these are very strong magnets and can break. Have one partner hold on to these until you are ready to attach them. The magnets should be placed close enough together so that the paperclip/washer hanging vertically down drifts to one magnet or the other if slightly disturbed from its equilibrium. Adjust the height as needed. Test the motion a bit and once you are satisfied that the magnets interact with the pendulum in a noticeable but not too extreme way, attach the magnets to the paper with tape. Assign each magnet a unique identifier by coloring the square it is in with a different color.

4. Describe the motion. Is the motion regular and predictable? Is there a well-defined period?

5. You should see that the nature of the motion depends on the location from which it is released. Your task is to identify those release points for which the motion is predictable and those release points for which it is not predictable.

6. Release the pendulum from above each of the squares marked on your paper. The pendulum should eventually come to rest above one of the magnets. If the pendulum consistently comes to rest above the same magnet when it is released from above the same square, mark that square with the label or color of that magnet. Otherwise mark the square as unpredictable.

7. Repeat this procedure for every square of the grid. You will eventually generate a pattern showing where the pendulum will end up when released. If you were to use a smaller and smaller grid your pattern would reveal increasing complexity on all scales. That is, it would be a fractal pattern. When you are finished, colour in the grid and compare your pattern with those of other members in the class. What are the similarities and differences?

Report

Your assignment is to write up your responses to the lab questions in the form of a short report in your lab notebook, and submit it with the map you created showing the eventual resting place of the pendulum when released. Your report should clearly contain well labelled tables with the data you recorded, with appropriate units. Your map should make use of colour. You should discuss your hypothesis and your results and state your conclusions.

Equipment

30 small rare earth magnets
meter stick
100 metal paperclips or small washers (ferromagnetic)
masking tape
15 stop watches
1 roll of massless string and scissors – (Dacron thread?)
1 ream of 11X17 paper
15 tall ring stands with, rods and clamps.