Relativity Workshop I: Space, Time, and Motion

1) You likely know that in relativity theory, the speed of light in a vacuum is special. The speed of light in a vacuum is 299792458 m/s exactly (we'll soon see how we can know the speed of light in a vacuum so exactly). This speed is so special and important that we represent it with its own symbol: *c*. For nearly all the calculations we will do, we'll usually round 299792458 m/s to 300000000 m/s.

a) Let's learn to "pronounce" these numbers. What do this mean? Well, we might say 2011 as "twenty eleven" but we might also pronounce it as "two thousand eleven". So, how do you pronounce 299792458? How do you pronounce 300000000?

b) It is convenient to represent 300000000 in scientific ("powers of 10") notation, where it would be given by 3×10^8. Why is 3×10^8 the same thing as 300000000?

2) Recall that distance, speed, and time are related by distance = speed x time. Use 3×10^8 m/s as the speed of light in a vacuum. It might help you to know that there are approximately 1609 meters in 1 mile.

a) In 1 second (1 s), how far does light travel (in a vacuum), in meters (m)?

b) How much is this distance (from part a) in miles?

c) Use your results from part a) and b) to determine the speed of light in miles per second.

d) How long does it take (in s) for light (in vacuum) to travel a distance of 1 m?

e) How far does light travel in 1 nanosecond (ns)? A nanosecond is 1 billionth of a second, which in scientific notation is written as 1×10^{-9} s. Express your answer in centimeters (cm).

f) Convert your answer from part e) to inches (there are 2.54 cm in 1 inch). How does this number compare to the length of standard sheet of paper (like this one)?

3) Some of the following calculations might be repetitive for you. Do as many as you need to develop confidence in this kind of units conversion, and to develop an intuitive sense of speeds, connecting numerical values to motion in the world. Please ask me if you're unsure of the third column. Use 3×10^8 m/s for *c*.

Speed (m/s)	Speed (mph)	Speed as fraction of speed of light	Notes
1.5	3.35	$5 \times 10^{-9} c$	This is five billionths the speed of light. Average human walking speed (with respect to ground)
30	67	$1 \times 10^{-7} c$	This is one ten-millionth the speed of light. Typical highway speed (with respect to ground)
240			Typical ground speed of commercial jet
600			(approximate) ground speed of the Concorde, not quite twice the speed of sound
3000			(approximate) ground speed of the X-43 scramjet plane
12000			(approximate) speed of Apollo 10 (w/ respect to ground) – record for manned vehicle
30000			(approximate) speed of earth in its orbit around the sun (with respect to the sun)
72000			(approximate) speed of the Helios-2 solar probe (with respect to the sun) – fastest man-made object

4) The following problem will continue to develop your idea of how large the speed of light is.

a) Fill in at least two rows in the following table, where you'll be converting distances into meters and miles. "1 lt·ns" is the distance light (in vacuum) travels in 1 ns, while "1 lt·yr" is the distance light (in vacuum) travels in 1 year.

distance (relativistic units)	**meters**	**Miles**	**Notes**
1 lt·ns			ns = nanosecond = 1 x 10^{-9} s = 10^{-9} s. That's one billionth of a second.
1 lt·s			In one second, light could travel a distance equal to about 7 times the circumference of the earth.
1 lt·min			The distance from the earth to the sun is approximately 8 lt·min.
1 lt·yr			The distance to Proxima Centauri (the nearest star that is not the sun) is approximately 4 lt·yr

b) What's the furthest distance you've ever travelled in one trip? How long did this trip take you (approximately)? How long would it have taken light to travel that distance?

5) Imagine that you walk on a perfectly flat plane at a constant speed.

a) Begin your walk by going due east a distance of 3 meters. Then walk due north a distance of 4 m. Now walk directly in a straight line back to where you started. Sketch your trip. How far did you walk, in total?

b) Assume that for each of the three parts of your walk, you walked at a constant speed of 2 m/s, and that you made the trip without pausing. How long did your total walk take?

c) Now, assume that the total walk actually took you 4 seconds. If you walked at a constant speed without stopping, how fast were you walking?

c) Sketch the points (1, 4) and (6, 16). What is the distance between these two points (assuming they are on a surface that is flat)? We haven't specified the units of distance in this coordinate system; if you'd like, you can call them meters. Or you could call them therblogs.

d) [CHALLENGE #1] Begin at the point (0, 0, 0) and travel in a straight line to the point (0, 4, 0). From (0, 4, 0) travel in a straight line to (3, 4, 0). From (3, 4, 0) travel in a straight line to (3, 4, 12). Sketch this journey as clearly as you can. If you were to travel in a straight line from the starting point at (0, 0, 0) to the ending point at (3, 4, 12), how far would you have traveled?

6) A train moves at 1.00 m/s with respect to the ground. You walk at a constant speed of 0.75 m/s (with respect to the train's floor). Assume that the ground and the train's floor are perfectly flat. You walk due east on the train, which is also moving due east.

a) Sketch this situation.

b) In 4 seconds, how far has the train moved (with respect to the ground) and in what direction?

c) In 4 seconds, how far have you moved (with respect to the train) and in what direction?

d) In 4 seconds, how far have you moved (with respect to the ground) and in what direction?

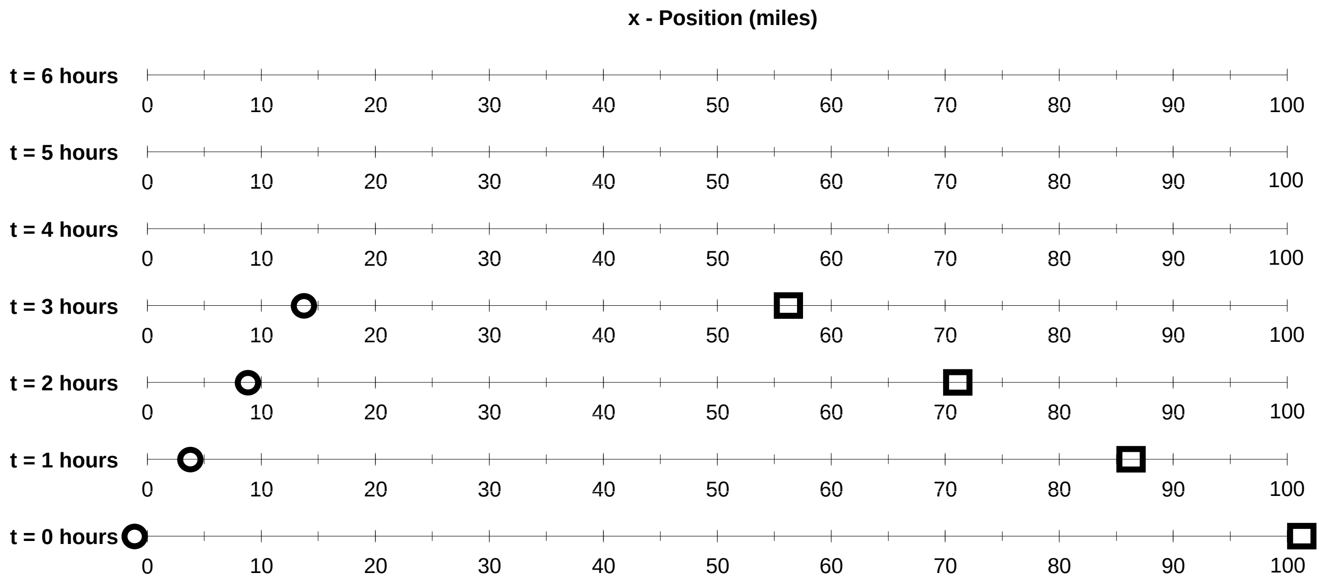
e) Now that you know the distance you traveled with respect to the ground and in what direction and for how long, determine your speed with respect to the ground, and the direction you are moving.

f) You might know a more direct way to determine your speed with respect to the ground. Describe and deploy this method, and compare to your answer from the previous part.

g) Now, you walk due **west** on the train (again at constant speed of 0.75 m/s with respect to the train's floor) with the train still moving due east at 1.00 m/s with respect to the ground. Using either of the methods above, determine your velocity with respect to the ground.

h) Now, you walk due **north** on the train (again at constant speed of 0.75 m/s with respect to the train's floor) with the train still moving due east at 1.00 m/s with respect to the ground. Sketch this situation. Determine your speed with respect to the ground.

7) Two (unoccupied) trains are on the same (straight, flat) track, moving towards each other. Train 1 is moving at some constant speed (to the right) and Train 2 is moving at some constant speed to the left (not necessarily the same speed as Train 1). The motion of the front of each train is shown in the diagrams below. The front of Train 1 is represented by the circle, while the front of Train 2 is represented by the square. Note that these diagrams are a modified version of the motion diagram you would get if you were to do video analysis on the trains. The main difference is that instead of all the "dots" being on the same picture, I've taken the pictures and stacked them vertically on this page (please ask me if that isn't clear), and I invented the name "stacked motion diagram". At t = 0 hours, the fronts of the two trains are 100 miles apart.

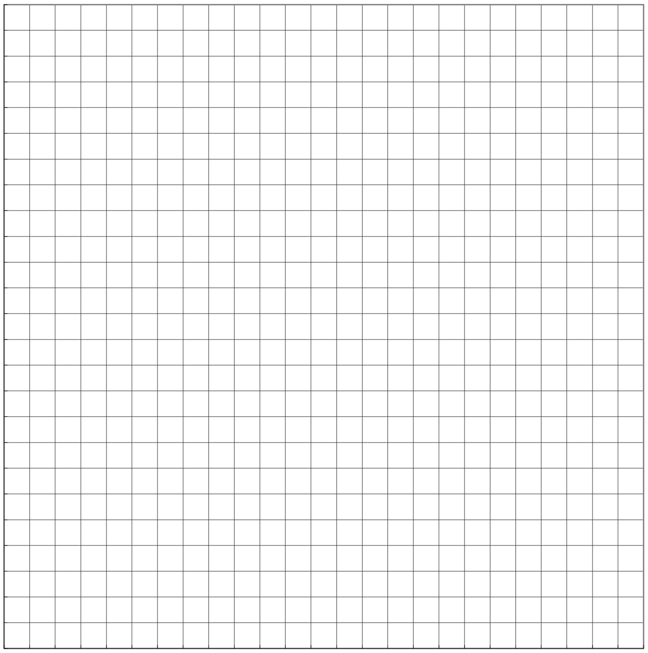


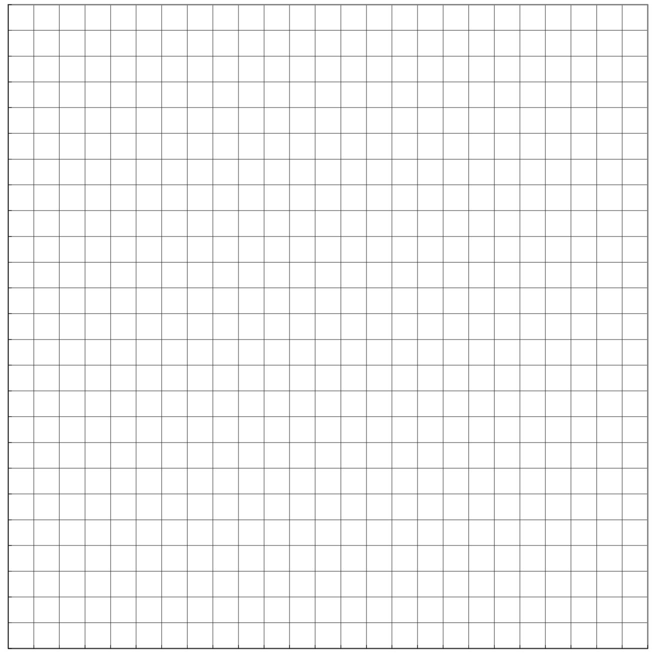
a) How fast is Train 1 moving? How fast is Train 2 moving? How do you know their speeds?
b) Each train is moving at constant speed. Fill in the rest of the stacked motion diagram for t = 4 hours, t =5 hours, etc. (You should make a judgment call about t = 6 hours.)
c) What is the relative speed of Train 2 with respect to Train 1? In other words, if you were sitting on Train 1, how fast would you measure Train 2 to be moving?
d) When do the trains collide?
e) Use the stacked motion diagram to fill out the following table:

time (hours)	x-position of first train (miles)	x-position of second train (miles)
0	0	100
1	5	85
2		
3		
4		
5		
6		

On the next page, you will find two gridded sections, intended for you to draw some careful graphs.

f) On the graph on the left of the next page, (labeled underneath as position vs. time graph), draw a position vs. time graph representing the motion of the two trains. Since the trains are moving at constant speed, you can connect the dots with a straight line. Recall our standard guidelines for graphs.
g) The graph on the right of the next page is labeled underneath as a spacetime diagram. A spacetime diagram is very much like a position vs. time graph, except it is a time vs. position graph. This means that time will be on the vertical axis and position on the horizontal. This is an unfortunately confusing convention that we'll have to learn to live with. Draw a spacetime diagram representing the motion of the two trains. You might notice that your spacetime diagram looks very much like the stacked motion diagram – they are essentially the same graph.





<u>position vs. time graph</u>

x-positions of trains 1 and 2 on vertical axis

time on horizontal axis

<u>spacetime diagram</u>

time on vertical axis

x-positions of trains 1 and 2 on horizontal axis

h) What do you notice about the slopes of the two lines on the position vs. time graph? Which is steeper? Does the steeper line correspond to the faster train or the slower train? Does this make sense?
i) Calculate the slope of the Train 1 line and the Train 2 line on the position vs. time graph. What physical quantity do these slopes correspond to?
j) What do you notice about the slopes of the two lines on the spacetime diagram? Which is steeper? Does the steeper line correspond to the faster train or the slower train? Does this make sense?

8) Challenge of the Superfly (optional, fun, difficult)

Enter Superfly, last fly from the doomed planet Flyon, rocketed to earth as an egg, there to grow up with powers far beyond that of mortal flies. We're most interested in Superfly's ability to fly in a straight line at 100 mi/h with respect to the ground, and to turn around instantly. Two trains facing each other on the same (straight) track are each traveling at 25 mi/h towards each other. At the instant that the fronts of the trains are 100 miles apart, Superfly begins to fly from the front of one train to the front of the other, traveling in a straight line at her top speed of 100 mi/h. Every time Superfly reaches the front of a train, she instantly turns around and travels in a straight line at 100 mi/h towards the other train. Eventually the two trains collide, trapping Superfly between them.

a) Sketch this situation on a position vs. time graph and/or a spacetime diagram as best you can.
b) How far has Superfly flown in total, traveling between the two trains starting from when they are 100 miles apart until they collide?
c) How many times has Superfly flown back and forth between the two trains?