1a. Gravitational size of a Black Hole

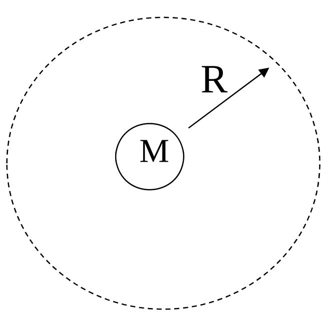
We can use energy conservation to find the size of a black hole. K = kinetic energy, U = potential energy. Find the escape velocity v from an object with mass M and radius R:

$$K_{initial} + U_{initial} = K_{final} + U_{final}$$
$$\frac{1}{2} mv^2 - GmM/R = 0 + 0$$

Solve for v^2=___

1b. Gravitational size of a Black Hole

For a black hole with mass M,
the escape velocity v=c at the
event horizon R = Schwartzschild radius



Find R in terms of G, M, and c.

(The exact GR calculation yields R/2 of the classical value we just derived.)

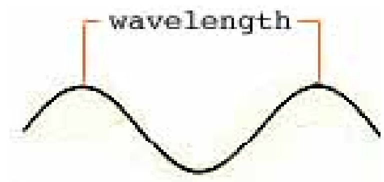
2. Quantum Mechanical size of a Black Hole

$$Energy\ E\ of\ photon \rightarrow wavelength\ \lambda\ of\ particle$$

$$E = \frac{hc}{\lambda} = pc \quad \rightarrow \quad momentum\ p = Mc = \frac{h}{\lambda}$$

$$Solve\ for\ wavelength\ \lambda\ \text{in}\ terms\ of\ M, c, h:$$

$$\lambda = \_\_\_\_\_\_\_\_\_\_\_\_$$



The deBroglie wavelength, λ, describes the smallest region of space in which a particle (or a black hole) of mass M can be localized, according to Quantum Mechanics.

3. Find the Planck mass, M_p

$$Schwartzschild\ radius = deBroglie\ wavelength$$

$$R_{gravity} = \lambda_{quantum}$$

$$\frac{GM_p}{c^2} = \frac{h}{M_p c}$$

$$Solve\ for\ the\ Planck\ mass:$$

$$M_p^{\ 2} = \_\_\_\_\_\_\_\_\_\_\_\_$$

If a black hole had a mass less than the Planck mass M_p,
its quantum-mechanical size could be outside its event horizon.
This wouldn't make sense, so M_p is the smallest possible black hole.

4. Find the Planck length, L_p

$$Substitute\ your\ Planck\ mass, M_p = \sqrt{hc/G}, into\ either\ R\ or\ \lambda:$$

$$R = \frac{GM_p}{c^2} = \underline{\qquad\qquad\qquad}$$

$$\lambda = \frac{h}{M_p c} = \underline{\qquad\qquad\qquad}$$

These both yield the Planck length, L_p. Any black hole smaller than this could have its singularity outside its event horizon. That wouldn't make sense, so L_p is the smallest possible black hole we can describe with both QM and GR, our current theory of gravity.

5. Calculate the Planck length and mass

$$Use\ these\ fundamental\ constants:$$

$$h \approx 6x10^{-34}\frac{kg\,m^2}{s},\ c \approx 3x10^8\frac{m}{s}\left(\tfrac{m}{s}\right),\ G \approx \frac{20}{3}x10^{-11}\frac{m^3}{kg\,s^2}$$

$$to\ evaluate\ the\ Planck\ mass, M_p = \sqrt{\frac{hc}{G}} = \underline{\qquad\qquad\qquad}$$

$$\text{and the Planck length } L_p = \frac{GM_p}{c^2} = \underline{\hspace{6cm}}$$

These are smallest scales we can describe with both QM and GR.

6. Calculate the Planck time

Consider the time it would take for light to cross the Planck length:

Speed = distance / time

$$c = L_p / \tau_p$$

Solve for the Planck time τ_p: