Patterning Math Lab 4

Part 1: Video Analysis of Two Dimensional Motion

You have seen that the Motion Detector is a powerful tool for measuring position vs. time (and calculating velocity vs. time) for one dimensional motion. However, the Motion Detector isn't good for measuring two dimensional motion. For that, video analysis is better suited. You have used video analysis to investigate motion in one dimension – the extension to two dimensions is straightforward. Here you will work with a partially analyzed video.

a) Open LoggerPro. Under File: Open, find the Sample Movies folder. In that folder, find the Basketball Shot folder. Open Basketball shot with analysis (**not** Basketball shot vector analysis).

b) Under Analyze, turn on Replay. Move the Replay window so that you can see both the movie window and the graph. In the Replay window, set the speed to 0.1 x Original. Then, press Start. Watch several times, comparing the motion diagram (the dots on the movie) to the position vs. time graphs. Set the speed lower if you'd like.

c) Examine the graph of X position and Y position vs. time. When (at what time) and where (X position and Y position) does the ball bounce off the ground? How do you know (what features on the graph did you use)?

d) Make a graph of X velocity and Y velocity vs. time. Use Insert: Graph; the new graph might already have Y velocity vs. time, so just add in X velocity (right click on the graph, choose Graph Options, under the Axes Option tab, scroll down the Y-Axis Columns and select X velocity). Resize and move the graphs so that the velocity vs. time graph is directly under the position vs. time graph and so that the horizontal axes match up (like the default setting with the motion detector). It is fine if this graph covers up the table.

e) Examine the X position vs. time graph and the X velocity vs. time graph while the ball is in the air and before it bounces. What does this tell you about the motion in the x direction, e.g. no motion, constant (non-zero) velocity, constant (non-zero) acceleration, other, etc.? What specifically do you use to make this conclusion?

f) Hopefully you concluded that the ball moved with constant velocity in the x direction. Determine the value of this constant velocity while the ball is in the air and before the first bounce. Record your results.

g) Now, examine the Y position vs. time graph and the Y velocity vs. time graph while the ball is in the air and before it bounces. What does this tell you about the motion in the y direction, e.g. no motion, constant (non-zero) velocity, constant (non-zero) acceleration, other, etc.? What specifically do you use to make this conclusion?

h) Hopefully you concluded that the ball moved with constant acceleration in the y direction. Determine the value of this constant acceleration while the ball is in the air and before the first bounce. Do this using both the Y velocity vs. time graph and also the Y position vs. time graph. Record your results. What remarkable numerical value do you obtain? Does this make sense?

i) Next, you will compare the motion diagram (the dots on the movie) with the Y position vs. time graph. They look superficially similar. Comment on the similarities and differences in appearances of the two.

j) Hopefully you noted that while the general shapes were similar, they didn't match exactly in the y-direction or along the horizontal axis. The y-direction can be taken care of by changing the vertical scale on the position vs. time graph (try it and see). Why won't changing the scaling on the horizontal axis also have a similar matching effect?

Part 2: Functions and Inverse Functions

This section of the lab focuses on the background reading from Ch 1.1, Ch 1.2, and Ch 1.6 of the precalculus text with the goal of better understanding functions and function inverses by using graphical techniques.

1. Read the definition of ***one-to-one*** function on p2 of Ch 1.1. Notice specifically that a one-to-one function is defined as the kind of function that can give exactly one output when used either forward or backward. Type in the linear function $f(x) = 0.5x - 1$ into Desmos. Use the graph to evaluate $f(4)$ and explain how to evaluate a function by following the graph. Now solve the function f at $f(x) = 4$. Explain how to reverse-evaluate (solve) a linear function by following the graph. Is the function f one-to-one? Explain how you know.

2. One-to-one functions always have inverse functions! It works the other way too -- if a function has an inverse, then the function must be one-to-one. Many times a function that is not one-to-one can be made into one that is one-to-one by restricting the range of the function (the set of output values). If a function can be made into a one-to-one function then an inverse function can be found. That's what we'll do now. Enter the equation $y=(x-1)^2$ into Desmos. Evaluate the equation at x = 3 by following the graph. Now reverse-evaluate (solve) the equation at y = 4. Does the equation represent a one-to-one function? Explain why or why not. Now put in a domain restriction {x>=0} (you may have to take the restrictions tour again which you can find using the Desmos ? in the upper right part of the Desmos screen). Now reverse-evaluate the equation at y=4. Does the resulting restricted equation represent a one-to-one function?
3. Starting with the function $y = (x-1)^2$ {x >=0} from the previous step, do some algebra and find an equation for the inverse function and plot it in Desmos. Be sure to define the inverse function with input variable x (Desmos doesn't like having y as an input variable). Plug in a few values to check that your new function correctly acts as the inverse.
4. Turn to Ch 1.6 p94 of your precalculus text and look at the graph in the lower right corner. Notice how inverse functions are mirror images around the line y = x. Examine your function and inverse function from the previous step and confirm that it has the symmetry around y=x.
5. Desmos can be used to quickly examine whether an equation determines a function or not. Read the precalculus text ch 1.1 p5-7. Do Exercise 5 on p13 of Ch 1.1.
6. Now do Exercise 13 on p15 of Ch 1.1.
7. Look at the toolkit functions on p11-12 of Ch 1.1. Which of the toolkit functions are one-to-one? Which aren't?
8. For each of the toolkit functions that are not one-to-one, give a domain restriction that will allow the function to have an inverse.
9. Are there any toolkit functions that cannot possibly have an inverse function?