Circle Trig Workshop

This workshop is an exploration of angles and trigonometric functions using graphical and analytical methods.

Materials: This workshop write-up, a Trig Table handout, graph paper, a handout with a 10 cm centered circle on graph paper, colored pencils, ruler, and protractor.

Part 1 - Measuring X and Y components of an angle on a circle of radius r
In this first part of the workshop we'll set up a table of x and y components of an angle using a protractor and ruler. In the last part of the workshop we'll use the trigonometric functions to fill in the x and y components.

1. Pull out your graph paper handout with the 10 cm circle. We'll be using this circle to fill in the table below of x and y components some angles. On the handout you'll see a coordinate grid with a circle centered at (0,0) with radius 10 cm.
2. You have colored pencils, so now is a good time to think about a color scheme for your workshop. Skim through the workshop to see what you'll be doing and pick some colors for the various segments you'll be drawing.
3. Using your protractor and ruler, draw a line segment between the origin and the circle at each of the angles in the table below. The x and y components of your angle are the x and y coordinates of the point where your angle segment meets the circle.
4. In an appropriate class notebook copy the table immediately below. For this first part of the workshop we'll only fill in the second and fourth columns for the measured x and y components of each angle.
5. Drop a perpendicular segment from each point on your circle to the horizontal axis. You should end up with a right triangle for each angle.
6. Measure the x and y components of each angle and fill in the x(measured) and y(measured) columns of the table. Don't worry about the 3rd and 5th columns yet.
7. Use the Pythagorean theorem to verify that your measured angle components correctly yield the radius of the circle.

θ (deg)	x (measured)		y (measured)	
53				
30				
60				
45				
90				
0				

Part 2 - Trigonometric tables

In this middle part of the workshop we'll develop the analytical techniques of trigonometry so we can apply the techniques to finding x and y components of angles. In particular, we'll start building a table of trigonometric functions that allows us to calculate lengths when we're given angles. The precalculus reading for this week covers these topics in depth. The purpose of the workshop is to engage in active reading and learning using the textbook ideas and techniques.

1. The x component of an angle is determined by the cosine (cos) of the angle and the y component of an angle is determined by the sine (sin) of the angle. In this part of the workshop we'll explore the sine and cosine functions ($y = \sin(\theta)$, $x = \cos(\theta)$) as they are viewed on a circle. We'll pick a few points on the circle, draw right triangles, and build a sine and cosine function table for a some angles using the Pythagorean theorem.
2. Bring out a fresh copy of the graph paper with the 10 cm circle. We'll be using this circle to fill in the trig table below.
3. In your notebook, build the following empty sine and cosine table.

θ (degrees)	cos(θ) exact	cos(θ) calculated	sin(θ) exact	sin(θ) calculated

4. Read Section 5.3 p321-322 of your precalculus text and produce a diagram for Example 1 on p 322, but double the radius and adjust the coordinates of the points accordingly. Your diagram should have the point (x,y) = (6,8) on the circumference of the circle, and a triangle as shown in the figure at the bottom of p321 of your precalculus textbook. For the purpose of this lab, let's call this triangle the *circle trig triangle* (CTT). (Mathematicians insist on naming everything and computer scientists make acronyms out of everything.)
5. Use the Pythagorean theorem to verify the correctness of the two sides and the hypotenuse of the triangle for Example 1 p322 of the precalculus text.
6. Use your protractor to measure the angle θ and fill in the angle measure in degrees in row one of the table.
7. In row one now fill in the exact cosine and sine values for the angle as a reduced fraction. Note that the value for the cosine of the angle is the x coordinate divided by the circle radius and the value for the sine of the angle is the y coordinate divided by the radius, as explained in the precalculus text. Leaving the cosine and sine as a fraction means the

values are exact.

The previous steps of the workshop constructed an angle from a convenient point, but the angle itself was not a convenient angle. In the remaining steps we will try to discover the sine and cosine values for more convenient angles.

8. On the circumference of your 10cm circle construct the point (x,y) with y = 5. Construct the CTT. You'll enter the data for this new angle in the second row of the table.
9. At this point you know the exact sin(θ) value as a fraction, so fill it in the next row of your angle table. Measure the angle θ and fill it in the table on the same row.
10. Use the pythagorean theorem and some algebra to calculate the cos(θ) value. Keep your value as a fraction with a square root so it is an exact value - don't use your calculator to obtain a numerical value yet. Put the exact value in the appropriate column.
11. Repeat the previous step of the lab for a point (x,y) on your circle with x = 5. By the end of this step you should have three angles in your table with sine and cosine values filled in.
12. There's another convenient angle that would be nice to have in the table - a 45 degree angle. Use your protractor to draw a 45 degree angle on your 10 cm circle and then make the CTT. Use the Pythagorean theorem to calculate the sine and cosine values of the 45 degree angle. Again, leave the values in square root form rather than converting them to numbers (with room for the number later). Fill in your table for the 45 degree angle.
13. Repeat the previous step of the lab for the two circle points (x,y) = (0,10) and (x,y) = (10,0). Note that you will not actually be able to build the CTT, but it should be obvious without the CTT what the angles are and what the corresponding sine and cosine values are for the angles. This should give you two more entries in your table. If you're confused about this, then look at Example 2 p322 in your precalculus text.
14. Now is a convenient time to fill in the calculated columns of your trig table. Use your calculator to compute decimal values for each of the cosine and sine entries and enter them in the table for each angle.
15. Finally, use the Trigonometric Tables handout to verify the cos(θ) and sin(θ) values for each angle in column one of your trig table. You can use your calculator also, but be sure you know how to put your calculator into degrees mode!

Part 3 - Calculating X and Y components of an angle on a circle of radius r

In this final part of the workshop we return to fill out the rest of the table of part 1 using analytical techniques of trigonometry developed in the second part.

1. Return to your first table above and label the third column with cos(θ) and label the fifth column with sin(θ).
2. Use the trigonometric table that you built in the previous step of this lab to fill in the cos(θ) and sin(θ) values for each row according to the angle in column one.
3. What is the mathematical relationship between the values in the x column and the values in the cos(θ) column?
4. What is the mathematical relationship between the values in the y column and the values in the sin(θ) column?

5. Finally, give a general formula for calculating the x and y components of any angle θ on a circle of radius r using the trigonometric functions.

By the way, you now know how to calculate the magnitude of the V_x and V_y components of any vector in physics because you know how to calculate the x and y components of any angle on a circle of any radius r !