Math Lab 9

In today's lab, you will explore closely some of the pre-calculus Ch 4 readings on exponential and logarithmic functions. You will also look at results from Physics Lab 14, investigate them mathematically, and gain some practical experience with exponential functions. You may not finish all of parts 3 and 4 in class, so finish the rest on your own – you'll find those parts of the lab helpful for some of the homework problems.

Part 1 - Graphs of exponential functions (Ch 4.2)

1. Enter the tables in Ch 4.2 p232 into Desmos. Don't connect the dots.
2. Figure out the pattern in the first table and propose a function f(x) to fit all the dots exactly. Plot your f(x) function in Desmos.
3. Figure out the pattern in the second table and propose a function g(x) to fit all the dots exactly. Plot your g(x) function in Desmos.
4. Is the function f(x) a growth or decay function? Is the function g(x) a growth or decay function?
5. Are either f(x) or g(x) undefined at any points in the domain?
6. Does the value of f(x) ever reach 0? Does the value of g(x) ever reach 0? Explain why or why not.
7. Specify both the formula and the domain and range for f(x) and for g(x).
8. Now enter into Desmos the parameterized function $h(x) = a*b^x$. Make both parameters sliders. The parameter a is called the *initial value* of the exponential and the parameter b is called the *base* of the exponential. What is the function h(x) when b is 1 (give a simple function definition based on what you see on the graph as you change the a parameter)?
9. Set the a parameter of h(x) to 1 and vary the b parameter to see how the graph changes. Pause when the h(x) graph lines up with your earlier f(x) and g(x) graphs. Do the values of the a and b parameters match up with those earlier functions?
10. Make a table in your lab notebook with column headers labeled something like the following. You'll fill in the table for various ranges of the a and b parameters in the next lab step. In the definition column put the simplest formula for the function. Leave room on the right for a few more columns that you may need below.

Range of parm a & b	h(x) simple definition or undefined	Domain	Range	Growth or Decay	y-intercept
a>0, b>1	$h(x) = a*b^x$				
a>0, b=1					
a>0, 0<b<1					
a<0, b<=0					
& etc.					

11. Using your table to record your observations, characterize the graph for various ranges of the value of the b parameter, that is, explain and record what happens when b>0, b=1, 0<b<1, b<=0. Start by fixing the a parameter at 1 and then varying the b parameter. Now set the a parameter to -1 and vary the b parameter through the three cases. Finally, set the a parameter to 0 and vary the b parameter through the three cases.
12. Does the a parameter affect the domain or range of the h(x) function?
13. Give a general form for the y-intercept point of (almost) every version of the h(x) function.* (Give a pair using the a and/or b parameters and/or constants).
14. Read p233 and Example 1 p233 and update your table above with any additional columns that you do not have. For example, you'll need a column with "long run behavior". Check your table against all the sample graphs in Example 1 p233. Use Desmos version of h(x) as needed for help or confirmation when checking your table.
15. Do Example 2 p235 without looking at the answer - cover up the lower graph in the example! This will test your knowledge up to this point.
16. Do the Try-it-Now on p235 by hand and then check your answers using your h(x) in Desmos.

17. Modify your definition of h(x) in Desmos to add a third parameter c that is the vertical shift. Read the second half of p236 to see how to do this. Set the values of the a and b parameters to some fixed values and experiment with the c parameter to watch the vertical shift.

Part 2 – Musical Mathematics

(If you did not participate in Physics Lab 14: Musical Mathematics download it from the Week 8 Calendar Page, find a reputable internet source that will give you the frequency associated with piano keys. Cite the source, including why you found it reputable. Use this to fill out the table below).

Physics Lab 14 (which continued the hands on investigation you began in Physics Lab 13) was a computer based activity in which you determined the fundamental frequency associated with keys 40 – 52 on a virtual piano keyboard.

"name"	key number	fundamental frequency (Hz)	first difference	second difference	ratio
C4	40				
C#4	41				
D4	42				
D#4	43				
E4	44				
F4	45				
F#4	46				
G4	47				
G#4	48				
A4	49				
Bb4	50				
B4	51				
C5	52				

1. Copy the following table into your lab notes and fill it out using your data from Physics Lab 14. Use the frequency column to calculate the first difference column, the second difference column, and a ratio column. If you don't remember how to calculate first differences and second differences, review your lab notes for Math Lab 2 and/or Math Lab 3. If you're not sure how to calculate the entries for the ratio column (this is new in this lab), consult with a neighbor or instructor. If you know how to use a spreadsheet, you can use that rather than using a calculator.
2. The fact that the ratio column is constant (or nearly constant given that this is measured data) means that the fundamental frequency vs. key number can be described by an exponential function. In Physics Lab 14, you should have seen that a linear model, a quadratic model, and a sinusoidal model were inadequate to properly describe the relationship between fundamental frequency and key number. Open up the corresponding LoggerPro file that graphs fundamental frequency vs. key number. If you don't have this LoggerPro file, make one now that shows a graph of fundamental frequency vs. key number.
3. Fit an exponential function to the data using the following: Under Curve Fit, use Base-10 Exponent (A*10^(Bx), (note that the x might be a different symbol depending on what/if you used for short name). Try Fit and if the fit looks good, hit OK, otherwise Try Fit again. Record your value for A and B.
4. Using exponent rules, show that you can rewrite A*10^(Bx) $= A \cdot 10^{Bx}$ as $A \cdot \left(10^{B}\right)^{x}$. If you're not sure how to do this, wait until you complete Part 3 and then come back to this.
5. Note that in Part 1 of this lab, you used $a \cdot b^{x}$ to describe an exponential function, where here you are using $A \cdot \left(10^{B}\right)^{x}$. Here, a and A are different symbols for the same quantity: the initial value of the exponential function. Convince yourself that $b = 10^{B}$ (this might really be as simple as you think).
6. Now, use your calculator and your curve fit value for B to determine a value for 10^{B}. Where have you seen this value? (Hint: look at your table from step 1, or see next Hint.)
7. Look at the frequency for C5 and compare it to the frequency of C4. C5 is one octave higher than C4, which means that C5 is twice the frequency of C4 (each octave doubles in frequency). Are your results consistent with this? The musical scale is set up so that each note is $2^{1/12}$ higher in frequency than the note before it, so that in 12 steps, the frequency doubles – thus, 12 keys to an octave. Calculate a numerical value for $2^{1/12}$. Where have you seen this value? (Hint: see previous Hint,)

Part 3 – Laws of Exponents and Logs

This section is a review of the textbook and lecture for the purpose of helping you absorb the laws of exponents and logs. Answer these questions in collaboration with your classmates. You'll want to have a copy of the handout with a list of the laws for exponents for reference.

1. Write down the law for multiplying numbers with exponents and give an example using integer exponents. What has to be the same in both of the numbers with exponents for law to work properly?
2. Write down the law for stacked exponents (e.g., like $(2^3)^6$)? What is the simplified form of $(2^3)^6$?
3. How do you write roots as exponents? Give two examples and then give a general form of translation.
4. There are more laws of exponents. Refer to the handout with a list of the laws for exponents. What is the law for dividing numbers with exponents?
5. Use the laws above to simplify the expression $(\text{sqrt}(5))^3$ * (sqrt(5)).
6. If you weren't able to do step 4 in Part 2, go back and try it now.
7. Find the page in Chapter 4 that lists five laws for logarithms (p253). For each of the laws, find an example in the text that makes use of that law primarily. Write down the law and the example number along with the page number in a table.
8. Read Examples 1 and 2 p243 in the textbook and then do the Try-it-Now on p243.
9. Read Examples 8 and 9 pp245-246 and then do the Try-it-Now on p245.
10. Do problem 4 in this week's problem set.
11. Do problem 7 in this week's problem set.
12. Do problem 19 and 20 in this week's problem set.

Part 4 - Solutions to exponential functions.

In this part of the lab we want to use Desmos to visualize and calculate solutions to problems modeled with exponential functions.

1. Read Ch 4.1 Example 1 p217 and use Desmos to graph a solution as an intersection of the function f(x) from the example and the vertical line x=12. In Desmos you'll have to use x, not t, in order to get the vertical line. Hover over the intersection to see that the solution matches the solution in the example. What is the simpler way to get the answer in Desmos?
2. Produce a formula and a graph for each of the three statements in the Try-it-Now on p217.
3. Use Desmos as a simple calculator to answer the question in the Try-it-Now on p218. Propose a more general question that could be asked about the two savings accounts in the Try-it-Now. How would you solve the more general question graphically? Analytically?
4. Do the Try-it-Now on p221 to find an exponential function given two points. Solve the problem by playing with your parameterized Desmos version of the exponential in Part 1 above ($h(x) = a*b^x$) and solve the problem analytically by following Examples 6,7 pp220-221).
5. Do problem 2 in this week's homework set.
6. Read Example 9 p223. Notice that this example is solving for an initial value. Propose a way to use Desmos to find the inital value. Hint - one approach is to treat the right side of the equation as a new function of a variable x that you'll put in place of the a parameter. Test your graphical solution in Desmos.
6. Do problem 11 in this week's homework set.
7. Determine the exponential function for problem 12 in this week's homework set and then answer the question using the Desmos calculator.