Patterning the World – Spring 2014 Name:____

Physics Lab 11: Vector Addition

Many physical quantities can be described by vectors. Today, we will work with vector addition of force vectors. We will combine three force vectors such that the resultant vector is equal to zero. When the forces acting on an object (in this case, a ring tied to three strings which are attached to weights hung on pulleys) add up to zero, we say that the object is in equilibrium.

Equipment: Force Platform, 3 legs, 3 pulleys, central ring with string (**note the care with which this is stored**), 3 mass sets (**keep track of each; don't mix them up**). Ruler and protractor (remember to return if you use a lab set). Work in groups of 3. Examine demonstration set-up and ask for assistance as needed to set up your station. Note: in this lab, g is for grams, not the acceleration due to gravity.

Getting Started:

a) Set up your Force Platform. For now, only use 2 pulleys.
b) Arrange 2 masses so that you have 100 g at 0° and 100 g at 180°. It should make sense that in this configuration, the ring will be in equilibrium: the tension in each string is equal (since the masses are equal) and they point in opposite directions (one tension pulls at 0° and the other tension pulls at 180°).
c) Now arrange 2 masses so that you have 100 g at 0° and 120 g at 180°. It should make sense that in this configuration, the ring cannot be in equilibrium. You will see that the ring is pressing against the center post; if you pull the ring away from the center post, it will accelerate in the direction of the unbalance force (in this case towards 180°) and press against the center post again.
d) Now arrange the 2 masses so that you have 100 g at 0° and 120 g at 90°. This should be another unbalanced state. This time, with your hand, pull on the third string in a direction such that the ring is in equilibrium. You should see that you are pulling in a direction that points in the third quadrant. You need to pull in a direction that compensates for the pull along 0° and the pull along 90°.

Example: In this example, we calculate what angle we need to put what mass in order to balance out 15 g at 0° and 20 g at 90°. Figure 1 shows a scale diagram of vector $\vec{A}$ (which has magnitude "15 g") and vector $\vec{B}$ (which has magnitude "20 g"). The scaling factor is 1 cm = 10 g. We want to find the vector $\vec{C}$ such that $\vec{A} + \vec{B} + \vec{C} = 0$. (Note that grams are a unit of mass and are not strictly speaking a unit of force, but the actual unit of force here is proportional to the mass.)

Figure 1 is a schematic representation of a bird's eye view of the Force Platform; the lengths of the arrows are proportional to the magnitude of the force (not the lengths of the string). We can see the center ring represented in Figure 1.

Figure 1

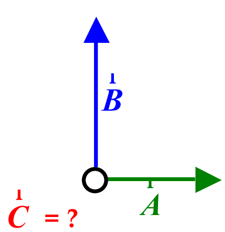


Figure 2

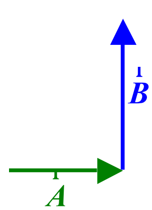


Figure 3

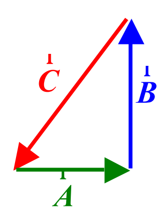
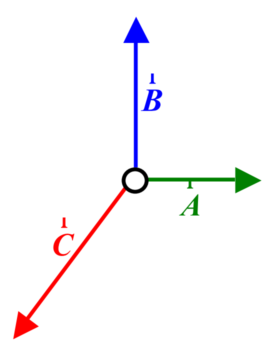


Figure 4



In Figure 2, we have rearranged vectors $\vec{A}$ and $\vec{B}$. As long as we don't change the length and direction of the vectors, this is a mathematically valid move. We see that $\vec{A} + \vec{B}$ does not equal 0, and can see what the vector $\vec{C}$ would need to be.

Figure 3 shows $\vec{C}$, and shows how $\vec{A} + \vec{B} + \vec{C} = 0$, as required. From this figure, we could measure the length of $\vec{C}$ and use the scaling factor 1 cm = 10 g to figure out the mass required. We can also measure the angle to find the direction of $\vec{C}$. Figure 4 returns to the schematic arrangement as in Figure 1, and shows each of $\vec{A}$, $\vec{B}$, and $\vec{C}$.

Vector diagrams are powerful and useful tools. Even more powerful is the analytical approach. The tables below summarize the results of the analytical approach.

	given information		→	calculated		
	magnitude (g)	direction (degrees)		*x* (g)	*y* (g)	note: if $\vec{A}$ and $\vec{B}$ had not been along easy directions, would need to use trigonometry (sine, cosine) to find components
$\vec{A}$	15	0	→	+15	0	
$\vec{B}$	20	90	→	0	+20	
$\vec{C}$	?	?	→	?	?	need to determine components of $\vec{C}$ in order to determine its magnitude and direction
$\vec{A} + \vec{B} + \vec{C}$	0	---		0	0	the vectors add up to zero because we are looking for equilibrium condition

	x	*y*	
$\vec{A}$	+15	0	can determine *x* and *y* components of $\vec{C}$ required to get to 0 in each column
$\vec{B}$	0	+20	
$\vec{C}$	**–15**	**–20**	
$\vec{A} + \vec{B} + \vec{C}$	0	0	

Use Pythagorean Theorem to get magnitude of $\vec{C}$:

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-15\,\text{g})^2 + (-20\,\text{g})^2} = 25\,\text{g}$$

Use trigonometry to get direction of $\vec{C}$:

$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) \pm 180^\circ? = \tan^{-1}\left(\frac{-20\,\text{g}}{-15\,\text{g}}\right) \pm 180^\circ?$$

$$= 53.13^\circ \pm 180^\circ? \text{ (from figure, yes } +180^\circ\text{)}$$

$$= 233^\circ$$

Try it yourself: For each of the 3 trials below, draw a vector diagram (to scale and as accurate as possible) and use the analytic approach to determine the angle and amount for the third mass in order to balance out the given 2. Test your result for each trial.

Trial 1: 90 g at 0°, 120 g at 90°. **Trial 2:** 50 g at 30°, 100 g at 135°. **Trial 3:** 100 g at 15°, 100 g at 135°.

Analysis:

a) You might have seen a symmetry that allowed you to take a short-cut in Trial 3. If you took the shortcut, go back and practice the full analytical method. If you didn't see the shortcut, look at your vector diagram as well as the results of your analytical method and see if you can figure it out.

b) In Trial 2, you had to resolve 2 vectors into components. What if instead you had rotated your vector diagram by 30° in the clockwise direction? Then one of the vectors would have been easy to resolve into components. Try this approach and see what happens. Note that this means the 100 g mass is now at 105° in the new coordinate system.

c) Look at the Trial 2 numbers again. Note that in some sense, you can use 50 g to balance 100 g + 100 g, in essence using 50 g to balance 200 g. How is this possible?