Math Lab 8

Today's lab is designed to support close reading of Ch. 8.2 and 8.5 in the precalculus text, to allow you to complete the associated problems, and to enable some open ended exploration. We hope that by the end, you will have investigated and created some amazing patterns.

Part 1 - Polar Coordinates and Equations
This part of the lab closely follows Ch 8.2 of the precalculus text, so open that section of your textbook (p.467).

1. In Desmos put in the cartesian coordinates on the unit circle for the common angles - see Ch 5.3 p.328 of your precalculus text or the unit circle you have produced for your notes. In Desmos you can enter cartesian coordinates as pairs in a list on a single line (e.g. (3,4), (5,6), etc.). Switch the coordinate system to polar (visit the wrench). Verify that in the polar coordinate system you have the polar coordinates for the common unit circle angles.
2. Follow Examples 1 and 2 p.467 and plot the points given in polar coordinates in that example. Desmos does not understand polar coordinates directly – there is not a way for it to differentiate (x, y) from (r, θ) So you'll have to convert the polar coordinates into cartesian coordinates (hint - use your knowledge of sine and cosine).* Stuck? Read ahead pp.468-469.
3. Use the Desmos polar coordinate system to check that you calculated the points in the previous step correctly. Now do the Try-it-Now on p.468. First calculate the cartesian coordinates.* Second, plot the points and check your answers against the polar coordinate system in Desmos.
4. Do the Try-it-Now on p.469 by hand*. Check your answers graphically.
5. Plot the polar equation in Example 5 p.470 graphically. Find three horizontal intercepts shown graphically. Find the three horizontal intercepts analytically (give exact values).* Propose a formula for all of the horizontal intercepts.*
6. Read Examples 6 and 7 pp.470-471. Input a version of the equation in Example 7 that uses a slider to control the amplitude parameter (rather than 4, have it be A with A a slider). Observe how the amplitude parameter affects the graph. Describe the effect(s) of the amplitude parameter on the graph.*
7. Predict what you will see if you switch the cosine in the equation of the previous step to a sine function - *do this before you try it in Desmos*.* To do the prediction, calculate by hand a table with a fey key values, and then by hand, plot the points. See Example 7 for the strategy.
8. Now check your prediction in Desmos. Did you predict correctly?*
9. Input the equation from Example 8, p.471. Adjust the input to be $r = 4\sin\theta + 2\{0 \le \theta \le c\}$. Change the range for c to be from 0 to 38 (note: 38 was chosen because it is close to 12π; for some reason Desmos limits the domain on theta in polar coordinates to have a maximum of 12π). Now, slowly increase the slider c. What do you observe? Here are a few relevant questions: where does the graph start? for what value of c (and thus theta) does it reach its maximum value (it might help to convert the decimal equivalent to a fraction of pi)? what is the maximum value? what are the values of c (and thus theta) where it crosses the x-axis (and thus correspond to horizontal intercepts)? Use this to work your way through the rest of Example 8, and briefly describe the main features of the graph and how it connects to its formula.*
10. Now, change the input to be $r \le 4\sin\theta + 2\{0 \le \theta \le c\}$ (you're changing the = to ≤). Starting with c = 0 as before and increase all the way to 38 (use the play button on the slider and decrease the play speed). What do you observe?
11. Plot a version of Example 8, p.471 using a slider k for the midline parameter. Use the value in Example 8 to check your equation. Vary k and describe the effect of that parameter on the graph.*
12. Type in the equation for the rose in Example 9 p.472. Read that example and see how a few simple calculations lets you predict by hand how the graph might look. Adjust your input to be $r = \cos(3\theta)\{0 \le \theta \le c\}$ using the same settings for c as earlier, and investigate what happens as you increase c from 0.
13. Do the Try-it-Now on p.473. Begin by hand calculating and hand plotting a table as in Example 9 before using Desmos.
14. Input $r = \sin(n\theta)\{0 \le \theta \le c\}$ and add in a slider for n. For now, set the range for n to be between 1 and 10. Also slide c to its maximum value of 38. Adjust n and see what happens to the pattern. The behavior can be complex; try just integer n to start. Use some of your other techniques, such as letting c vary and changing = to ≤.
15. Repeat the previous, but this time set the range for n to be between –1 and 1.
16. Use Desmos to do problems 37-48 on p.478.
17. Look at problems 49-60 on p.479 and see if you can predict what the graph would look like from your experience and

your earlier predictions about the effects of the various parameters in the generalized polar equation. Test your predictions.
18. Use Desmos to graph the polar equations in problems 61-66 on p.479.

Part 2 - Parametric Equations
This part of the lab closely follows Ch 8.5 of the precalculus text, so open that section of your textbook (p.504).

1. Do the parametric equations of Example 2 p.506 by typing in the following pair, making sure to use the parenthesis: (t^2+1, 2+t). The first component of the pair is the equation for the x values as a function of t. The second component of the pair is the formula for the y values of the pair.
2. Do the parametric equations of Example 3 p.506 by typing in the following pair:
((2cos(t), 3sin(t)).
3. Input ((Acos(t)),(Bsin(t))) and have sliders for A and B. Play with the sliders and describe the effect of each of the sliders on the graph.*
4. Do the Try-it-Now on p.507. As before, put the x formula in the first element of a pair and the y formula in the second element of the pair.
5. Input $\left(A\cos\left(vt\right), B\sin\left(wt\right)\right)$ with sliders for A, B, v, and w. Play with the sliders and observe the results. Make a prediction about the effect of each of the parameters.* Hint - this is another complex figure so you might investigate the effects of changing just A and v or B and w.
6. Do the Try-it-Now on p.509. If needed, read the preceding discussion on parametrizing equations. Plot the original equation in Desmos along with your parametric version to make sure they give the same curve.
7. Plot the parametric equations of Example 11 p.511 in Desmos.
8. Plot the parametric equations of Example 12 p.512 in Desmos. Now generalize Example 12 by putting in sliders for the amplitude and the angular frequency. Use the same amplitude slider in both formulas and the same angular frequency slider in both formulas so you can keep a symmetrical figure. Play with the parameters and observe the results. Compose a statement about the effect of each of the parameters on the graph.*
9. Do the Try-it-Now on p.512 and display your results in Desmos.
10. Do problem 42 on p. 517 analytically as in the problem and also graphically using Desmos.

Part 3: Open Investigations
1. Input $r = A\sin\left(\frac{a}{b}\theta\right) + k\{0 \le \theta \le c\}$ with sliders for A, a, b, k, and c (and set the range for c as before). Investigate the effects of changing each parameter A, a, b, and k independently and together. Use techniques you have developed in earlier parts (especially changing c and changing = to $\le$) to see if you can begin to make some sense of the complexity that you observe.
2. For some amazing displays, hit the play buttons on all the sliders, and change the play speeds to different values.
3. Open the following Desmos graph **https://www.desmos.com/calculator/a0et2fqiog**
"a" is the radius of the inner circle, "b" is the radius of the outer circle. Begin by picking some a and b that you like. The defaults are fine. Hit the play button on the w slider (input box 4). After observing a full cycle and watching the path traced out by the red dot (you don't need to let w run the full time), see if you can draw the path the red dot has traced out. Then, set w back to 0, and turn on the graph displayed in input box 5. Now, hit play on w again. Play with this by changing the a and b values.