Physics Lab 7: SuMarble Monday!

Goals: Improve communication and teamwork capacities**;** Improve ability to make careful measurements; Use constant acceleration kinematics equations and independence of perpendicular motion to model the motion of a marble (and other objects) in two dimensions.

Equipment: You will be oriented to the location and proper use of the equipment for this lab. At the end of the session, return the equipment to its original configuration and location.

Today's activities continue and extend Physics Lab 6.

Constant acceleration formulas:

$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \Rightarrow \Delta x = v_{0x}t + \frac{1}{2}a_x t^2$	$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \Rightarrow \Delta y = v_{0y}t + \frac{1}{2}a_y t^2$
$v_x = v_{0x} + a_x t \Rightarrow \Delta v_x = a_x t$	$v_y = v_{0y} + a_y t \Rightarrow \Delta v_y = a_y t$
$(v_x)^2 = (v_{0x})^2 + 2a_x \Delta x$	$(v_y)^2 = (v_{0y})^2 + 2a_y \Delta y$

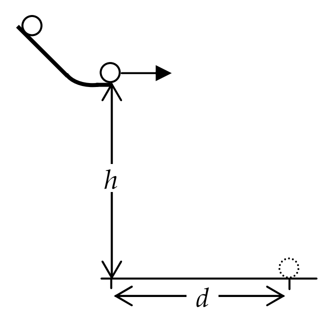
Special case: projectile motion → free fall (constant acceleration in vertical direction) + constant velocity in horizontal

$a_x = 0$	$a_y = -g = -9.8\ \text{m/s}^2$ (standard coordinate system)
$v_x = v_{0x}$	$y = y_0 + v_{0y}t - \frac{1}{2}gt^2 \Rightarrow \Delta y = v_{0y}t - \frac{1}{2}gt^2$
$x = x_0 + v_{0x}t \Rightarrow \Delta x = v_{0x}t$	$v_y = v_{0y} - gt \Rightarrow \Delta v_y = -gt$

Forms of quadratic functions:

standard form:	vertex form:	intercept (factored) form:
$f(x) = ax^2 + bx + c$	$f(x) = a(x-h)^2 + k$ vertex: (h,k) $h = -\frac{b}{2a}$ $k = c - \frac{b^2}{4a} = f(h)$	$f(x) = A(x - x_L)(x - x_R)$ x_L, x_R are roots or x-intercepts

Part I: Horizontal Launch



Getting Started
Gather your equipment and assemble it as described. Determine a release point so that when the marble falls a vertical distance *h* of about 0.30 m, it travels a horizontal distance *d* of about 0.20 m. Always release the marble from rest from that point.

Investigation 1: Launch speed of marble

a) Using foil taped to the bench surface, measure *h* and *d* as carefully as possible. Record these results.

b) Since you have launched the marble horizontally, what is v_{0y}, the initial velocity in the vertical direction (here initial means just when the marble is launched/leaves the ramp)?

c) Why does $\Delta y = v_{0y}t - \frac{1}{2}gt^2$ simplify to $\Delta y = -\frac{1}{2}gt^2$ in this case?

d) Why does $\Delta y = -h$ (in other words, why is Δy a negative number)? Explain both in terms of the choice of coordinate system (what did you call your origin?) and also in terms of the equation $\Delta y = -\frac{1}{2}gt^2$ (what happens mathematically if Δy is a positive number?)

e) Since $\Delta y = -h$ and you have measured *h* and $\Delta y = -\frac{1}{2}gt^2$ where *g* = 9.8 m/s², you can solve for *t*, the time it takes the marble to fall from the ramp to the ground (table). Determine *t*.

f) In this model, the acceleration in the vertical direction was constant. What constant was it? Why? What assumptions were made?

g) In this model, the acceleration in the horizontal direction is zero. What assumptions were made? Why does this mean that the horizontal velocity is constant?

h) Since $a_x = 0$, the horizontal velocity is constant, and $\Delta x = v_{0x}t$, so $v_{0x} = \frac{\Delta x}{t}$. Which of your measured quantities equals Δx? Do you know *t*? Calculate v_{0x}. Why is this the launch speed of the marble?

Investigation 2: Predict an impact location

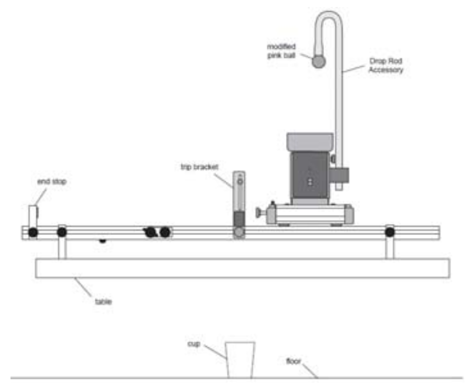
a) Raise the ramp **as high it will go on the ring stand (at least 0.50 m)**. Measure the height *h* as carefully as possible.

b) Why is the launch speed in this case the same as the launch speed you calculated previously?

c) Use your previously determined launch speed and the current height to calculate the horizontal distance the marble will travel before hitting a vertical surface. In other words, predict the impact location. Follow the same chain of reasoning as before: use the vertical direction to determine the time in the air (this will be the exact same calculation as before, except with your new *h*. Why?). Then, use $\Delta x = v_{0x}t$ (why can you use this?).

d) With an instructor present, test your prediction.

Part II: Cup Catch



a) This is a slightly different version of **Part I**, but with the same essential physics and math.

b) Follow the instructions on the board to determine the horizontal speed of the cart on the track. Why is this also the horizontal speed of the pink ball?

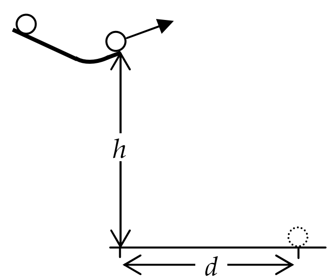
c) Recall that the pink ball is released when the cart passes the trip bracket. Your goal is to determine where on the floor relative to the trip bracket you should put the cup so that the pink ball lands in the cup. What will you need to measure? Be specific.

d) If the apparatus is occupied, move on to **Part III** until it becomes available. Otherwise, make your measurements.

e) Using your measurements, determine where you should put the cup.

f) **This part requires instructor supervision**. If the apparatus is available, test out your prediction. Place your cup in your calculated location, and see what happens.

Part III: Angle Launch



a) So far, we have limited ourselves to cases where the initial velocity was purely horizontal. What happens if the initial velocity has components in both the x- and y-directions (what simplifications no longer hold?)

b) Angle the ramp so that the launch velocity is at an angle (with respect to horizontal) somewhere between 10^{o} and 45^{o}. You will need to experiment to make sure you can still get a decent launch speed; you may need a different release point – if so, mark the new release point.

c) Carrying out an investigation as in Part I would be very challenging using our previous methods. We don't know the launch speed nor do we know the launch angle, so actually it is not possible to solve with our previous methods. Even if we did know the launch speed **<u>or</u>** the launch angle, it would be quite challenging mathematically (I invite those of you who are interested to take on this challenge – it is possible to measure the angle of the ramp well enough, and then, by measuring *h* and *d*, determine the launch speed). If we knew the launch speed and the launch angle, this becomes possible to do with the physics and mathematics we have been using, though still somewhat challenging. **Instead, we will take advantage of the pattern we have observed for this motion and the mathematical structure used to describe this pattern: parabolas and quadratics.**

d) What is the minimum number of points required to determine an equation for a parabola? Do you know why?

e) Note that we automatically have one point of this parabola: the origin. This means we need just 2 more (for the previous question, did you answer that you need 3 points to determine an equation for a parabola?). For your 2nd point, set *h* to be approximately 10 cm (and measure *h* as carefully as you can) and determine *d*. For your 3rd point, set *h* as high as you can given your ring stand (at least 50 cm, and measure it as carefully as you can), and determine *d*. Copy the data table to your notebook and fill it out, changing the *h* values to match your investigation. For now, leave the interpolated and extrapolated rows blank.

	h (cm)	d (cm)	$x = d$ (cm)	$y = -h$ (cm)
1st point	0	0		
2nd point	~10			
3rd point	~50			
interpolated				
extrapolated				

f) In the data table, why do $x = d$ and $y = -h$? Does this have anything to do with where we chose the origin?

g) Review the standard form for a quadratic function. Why would we be interested in a quadratic function in this case? The standard form has $y = ax^2 + bx + c$, with unknown parameters *a*, *b*, and *c*. Three parameters is a lot to solve for. However, we are lucky in this case. One of our points is the origin, which simplifies our quadratic function by quite a bit, reducing it from 3 unknown parameters to 2. Show that since the origin is one of the points on the parabola, $y = ax^2 + bx + c$ simplifies to $y = ax^2 + bx$. In principle, you should also be able to get values for *a* and *b*; we'll leave this for the Analysis.

h) Launch LoggerPro. Enter your *x* and *y* data for the first three points into the table. The program should automatically plot those points as well as rescale the axes to show you all three points. Note that in contrast to graphs we have frequently constructed, this is not a position vs. time graph: this is a *y* vs. *x* graph.

i) Highlight the 3 points, and fit a quadratic (you may need to change the horizontal scaling so that it starts a little before *x* = 0 and goes a little after your largest *x* value in order to highlight all 3 points). Write down the best fit quadratic function using the fit values for A, B, and C. Based on your answer to h) above, what should C be? (Note: in LoggerPro, a number provided as 1E-15 is how the program represents $1x10^{-15}$. Compared to the other numbers for A and B, this is basically zero.)

j) Our next goal is to **predict** where the marble will land for *h* = 25 cm (again, doesn't have to be exactly 25 cm but you must measure *h* as carefully as possible). Set your ramp to the appropriate *h*, measure it carefully and record this value in the interpolated row on your table. **Don't launch the marble yet.**

k) In principle, since we have the quadratic function that we believe models the parabolic trajectory of the marble, we can solve this for $y = -h$ to determine x and thus predict the impact distance. Write down the equation you would solve, substituting all the numerical values for *h*, *A*, *B*, and *C* (well, using *C* = 0 is fine), leaving an equation whose only unknown is *x*. This can be solved for *x* in a number of (related) ways. We'll leave that analytic approach for the Analysis.

l) Launch Desmos. Create a table with the *x* and *y* columns from your data table as the *x* and *y* columns in the Desmos table. Change the graph scaling so you can see the data points on the graph (Desmos does not autoscale like LoggePro.) Create the function y = Ax^2 + Bx using your values for A and B from the LoggerPro quadratic fit. Verify that the function goes through the data points.

m) By clicking and dragging on the function graph, you can get the coordinates for any point. Find the x coordinate the corresponds to the $y = -h$ you set on the ramp. That is your predicted impact location. Fill out the rest of the interpolated row in your data table.

n) Test your prediction!

o) If you have time, repeat for an extrapolated height. Move the ramp to the edge of the lab bench so that the marble will land on the floor. Measure *h* as carefully as you can, and record that in the extrapolated column of your data table. Following the same process as in m), determine the impact location and record it in your data table. **Test your prediction!**

Analysis

a) Given the x and y coordinates for the first 3 points in your data table, determine the quadratic function mathematically, and compare to your results from the LoggerPro fit.
b) Given the quadratic function you determined (through LoggerPro or analytically), solve for your interpolated $y = -h$ value to determine x, and compare to your graphical results via Desmos.

Implications

Though the investigation you have completed may not seem profound, the implications are. Let's consider several (some of these are perhaps more philosophical than others, but I encourage you to think about and discuss them all):

a) What does the fact that your predictions were correct mean about your assumption that the acceleration in the y-direction was 9.8 m/s^2 down? What does this mean for our assumption about neglecting air resistance?
b) What does the fact that your predictions were correct mean about your assumption that the acceleration in the x-direction was zero? What does this mean for our assumption about neglecting air resistance?
c) What does the fact that your predictions were correct mean about your assumption that the motion in perpendicular directions was independent?
d) What does the fact that your predictions were correct mean about the validity of your mathematical model?
e) What does it mean that a mathematical model can describe natural phenomena so carefully that you can effectively predict the future?