# Math Lab 9

In today's lab, you will look at results from the past few Physics Labs and investigate them mathematically, specifically gaining more experience with power functions and exponential functions. You will also take a big picture view of the quarter, looking for connections between natural phenomena you have observed and measured in the context of the main mathematical functions we have studied (this is described on the front board).

**Part 0 – When and Where to go and be on Wed. March 5**
Go to the Week 9 Calendar page and under Wed. March 5, follow the link "Check here to see where you should be during Physics lab time." Carefully note down when and where you should be (your time in physics lab that day is split up into two slots that meet in two different places). Note that if you miss the scheduled Lab Clean-up, you will have to complete a separate (and likely more difficult) lab clean-up on your own time which you will need to schedule with Diane which takes up her time.

**Part 1 – Pendulum Periods**
(If you did not participate in Physics Lab 12: Pendulum Periods, move on to Part 2.)
Physics Lab 12 (which continued in between data collection associated with Physics Lab 13) had you investigate how the period of a pendulum was affected by the mass of the bob, the amplitude (angle) of the swing, and the length of the pendulum. In Galileo's Dialogue Concerning Two New Sciences, he made the following claims based on his investigation of how these factors affected the period of a pendulum:
period is independent of mass;
period is independent of amplitude;
period is dependent on length, with the square of the period proportional to the length.

1. Using LoggerPro, make a table with mass and period columns, enter your data, and make the associated graph of period vs. mass (LoggerPro may make the graph automatically). Make sure to have the columns labeled appropriately (right click on the column heading, choose Column Options, pick the appropriate column and change the Name, Short Name, and type in appropriate units). As needed, you can make a new column under Data: New Manual Column. Based on your data and graph, are your results consistent with Galileo's claim that the period of a pendulum is independent of mass?
2. Examine your data for amplitude and period. As needed, make graph(s) in LoggerPro. Based on your investigation, what would you conclude about Galileo's claim that the period of a pendulum is independent of amplitude? Can you modify Galileo's claim to be consistent with the results of your investigation?
3. Using LoggerPro, make a table with length and period columns, enter your data, and make the associated graph period vs. length. and graph of period vs. length. Follow similar steps as 1. above for naming columns. Based on your data and graph, are your results consistent with the first part of Galileo's claim: that the period is dependent on length?
4. Let's investigate further Galileo's claim that the square of the period is proportional to the length. Mathematically, this would be represented as $P^2 = cL$ , where $P$ is the period, $L$ is the length, and $c$ is the proportionality constant. Show that this can be re-written as $P = \sqrt{cL} = (cL)^{1/2} = c^{1/2}L^{1/2}$. Note that this means that $P$ is a power function of $L$, with power ½.
5. Use LoggerPro to find a power function fit to your data. Highlight the data, and under Curve Fit, choose Power (Ax^B, but depending on what you short named your column, the x might be a different symbol). Click Try Fit, and if the fit looks, good, select OK. If the fit doesn't look good, hit Try Fit again. Write down the values for A and B from the Curve Fit. Is your value for B (the power in the power function) consistent with Galileo's claim?
6. Actually, we know the proportionality constant. The period of a pendulum for small angle swings is given by $P = \sqrt{\frac{4\pi^2 L}{g}} = \frac{2\pi}{\sqrt{g}} L^{1/2}$ where $g$ is the acceleration due to gravity, 9.8 m/s$^2$. This means that the curve fit value A should equal $2\pi / \sqrt{g}$ . Calculate a numerical value for $2\pi / \sqrt{g}$ and compare it to your curve fit value for A.
7. What do you conclude about Galileo's claim that the square of the period is proportional to the length?

**Part 2 – Musical Mathematics**

(If you did not participate in Physics Lab 11: Musical Mathematics download it from the Week 8 Calendar Page, find a reputable internet source that will give you the frequency associated with piano keys. Cite the source, including why you found it reputable. Use this to fill out the table below).

Physics Lab 11 (which continued the hands on investigation you began in Physics Lab 10) was a computer based activity in which you determined the fundamental frequency associated with keys 40 – 52 on a virtual piano keyboard.

| "name" | key number | fundamental frequency (Hz) | first difference | second difference | ratio |
|---|---|---|---|---|---|
| C4 | 40 | | | | |
| C#4 | 41 | | | | |
| D4 | 42 | | | | |
| D#4 | 43 | | | | |
| E4 | 44 | | | | |
| F4 | 45 | | | | |
| F#4 | 46 | | | | |
| G4 | 47 | | | | |
| G#4 | 48 | | | | |
| A4 | 49 | | | | |
| Bb4 | 50 | | | | |
| B4 | 51 | | | | |
| C5 | 52 | | | | |

1. Copy the following table into your lab notes, and fill it out using your data from Physics Lab 11. Use the frequency column to calculate the first difference column, the second difference column, and a ratio column. If you know how to use a spreadsheet, you can use that rather than using a calculator.
2. The fact that the ratio column is constant (or nearly constant given that this is measured data) means that the fundamental frequency vs. key number can be described by an exponential function. In Physics Lab 11, you should have seen that a linear model, a quadratic model, and a sinusoidal model were inadequate to properly describe the relationship between frequency and key number. Open up the corresponding LoggerPro file that graphs frequency vs. key number. If you don't have this LoggerPro file, make one now that shows a graph of frequency vs. key number.
3. Highlight the data, and fit an exponential function to the data. Under Curve Fit, use Base-10 Exponent (A*10^(Bx), note that the x might be a different symbol depending on what/if you used for short name). Try Fit and if the fit looks good, hit OK, otherwise Try Fit again. Record your value for A and B.
4. Using exponent rules, show that you can rewrite A*10^(Bx) $= A \cdot 10^{Bx}$ as $A \cdot \left(10^{B}\right)^{x}$. Use your calculator and your curve fit value for B to determine a value for $10^{B}$. Where have you seen this value? (Hint: look at your table from step 1.)
5. Look at the frequency for C5 and compare it to the frequency of C4. C5 is one octave higher than C4, which means that C5 is twice the frequency of C4 (each octave doubles in frequency). Are your results consistent with this? The musical scale is set up so that each note is $2^{1/12}$ higher in frequency than the note before it, so that in 12 steps, the frequency doubles – thus, 12 keys to an octave. Calculate a numerical value for $2^{1/12}$. Where have you seen this value?

**Part 3 – Keeping it Cool**

(If you did not participate in Physics Lab 13, move on to the activity described on the board)

In Physics Lab 13, you investigated cooling rates of hot water under varying conditions. Newton modeled the cooling process by assuming that the rate at which thermal energy moved from one body to another is proportional (by a constant *k*) to the difference in temperature between the two bodies, $T_{diff}$. In the case of a sample of water cooling in room temperature air, the cooling rate = $-kT_{diff}$ .

From this simple assumption he showed that the temperature change is exponential in time and can be predicted by

$$T = T_0 e^{-kt} + T_{surroundings}$$

where $T_0$ is the initial temperature difference. When a rate of change is proportional to the changing quantity the behavior is exponential.

1. Open up your saved LoggerPro file, and display graphs of Cooling 1, Cooling 2, and Cooling 3 (these can be separate graphs or on the same graph). Use Curve Fit to fit the Natural Exponential function (A*exp(–Ct)+B) to the data for each cooling curve.

2. Match the variables A, B, and C in the fitted equation to terms $T_0$, $T_{surroundings}$, and *k* in the expression of Newton's Cooling Law.