

Allman Ch 3.4 Prob 3 and 4

- 3.4.3. Since $r = u$ and $s < v$ for the first choice of parameters, we expect the immune system to have the advantage. Simulations show it typically 'wins', eliminating the infectious agent, though initially the infectious agent may grow. For the second choice of parameters, we expect the infectious agent to have the advantage. It typically 'wins' in simulations by growing to infinity, unless initially the immune agents are much more plentiful. The first set of parameters gives a more desirable outcome if your own immune response is being modeled.
- 3.4.4. The P -nullcline is the P -axis and the vertical line $P = r/s$. The Q -nullcline is the P -axis and the vertical line $P = u/v$.
For most parameter choices, the equilibria are all points on the P -axis. If $r/s = u/v$, then all points on the vertical line $P = r/s$ are also equilibria. To the left of the line $P = u/v$ arrows point up, and to the right of it they point down. To the left of the line $P = r/s$ arrows point to the right, and to the right of it they point left. This model can be in equilibrium only if the infectious agent is eradicated, but any amount of immune agent may remain.
For $u/v < r/s$ we might expect orbits to typically move toward an equilibrium. Small amounts of infectious and immune agents might grow for a bit, and then the immune agents might continue to grow while reducing the infectious ones toward 0.
For $u/v > r/s$ we might expect orbits to typically move toward $P = r/s$ as Q goes to infinity.
However, these scenarios are not guaranteed, as orbits may 'jump' by amounts that the direction arrows are not sufficient to predict.