

1. Consider the following game modeling the interaction between two individuals that we considered in the week 4 workshop. In this type of interaction there is a task to be completed (eg building a nest, or dam), from which they both get benefit  $b$ . If they both work to complete the task they each incur a cost  $c/2$ , but if only one does the work that individual incurs cost  $c$  and the other incurs no cost. There are two strategies: Work and Shirk.
  - (a) Write down the payoff matrix for this interaction game and determine under what conditions on  $b$  and  $c$  this is a prisoner's dilemma game. What do cooperation and defection correspond to in this game?
  - (b) Suppose two individuals repeat this game  $m$  times. Find the payoff matrix for a T4T player playing an AllD player and determine the conditions on  $m$  for T4T to be an ESS? Express your answer in terms of  $b$  and  $c$ .
  - (c) If the conditions are right for both AllD and T4T to be ESS, then a mixed population of players will evolve towards one or the other strategies depending on the initial distribution of strategies. The separation between those tendencies comes at the unstable equilibrium distribution. Let's take  $c = 6$  and  $b = 4$ . Suppose there is a population of  $x$  T4T players and  $1 - x$  AllD players. Find an expression for the fitness of T4T and AllD players in this population as a function of the number of times they play and the proportion  $x$ . Hence find the unstable equilibrium value of  $x$  as a function of  $m$ .
  - (d) If there is initially an equal distribution of AllD and T4T players, what is the minimum value of  $m$  so that the system evolves to one in which everyone plays T4T?
2. Given the repeated prisoner's dilemma game with payoff matrix

$$\begin{pmatrix} 3 & 0 \\ 5 & 1 \end{pmatrix}$$

consider the set of strategies which are contingent on what the players chose to do in the previous round. There are 16 strategies. We can write each strategy as a list of length four containing ones and zeros, with a one meaning cooperate and a zero meaning defect. The first entry indicates what to do if the previous round was  $CC$  (ie I cooperated and so did my opponent). The second entry is what to do if the previous round was  $CD$  (ie I cooperated but my opponent did not). The third is  $DC$  and the last is  $DD$ . For example, T4T is  $(1, 0, 1, 0)$ . For each of the following strategy pairs write out the sequence of moves they each make they play each other in an 8 round game and give the corresponding payoffs. Assume player  $A$  starts by cooperating and player  $B$  starts by defecting. Then find the  $2 \times 2$  payoff matrix for the 8 round game and find the ESS's.

- (a)  $A : (1, 0, 1, 0)$  and  $B : (0, 1, 0, 1)$
  - (b)  $A : (1, 0, 0, 1)$  and  $B : (1, 0, 1, 0)$
3. Given the general prisoner's dilemma matrix

$$\begin{pmatrix} R & S \\ T & P \end{pmatrix}$$

- (a) Show that Pavlov (ie  $(1, 0, 0, 1)$  ) is stable against invasion by AllD provided

$$R > \frac{(T + P)}{2}$$

- (b) Challenge for advanced math students: If the above condition does not hold then show that the stochastic strategy  $(1, 0, 0, p)$  is stable against invasion by AllD provided

$$p < \frac{(R - P)}{(T - R)}$$