

Allman Ch 2.1 1,4,6,9

2.1.1. a. $\begin{pmatrix} 0 \\ 17 \end{pmatrix} = (0, 17)$
b. $(-1, 11, -18)$
c. $\begin{pmatrix} 0 & -8 \\ 17 & 30 \end{pmatrix}$
d. $\begin{pmatrix} -1 & -2 & 7 \\ 11 & 7 & -8 \\ -18 & -1 & -1 \end{pmatrix}$

2.1.4. a. $\begin{pmatrix} 4 & 2 & -2 \\ 0 & 1 & 2 \\ -1 & 0 & 1 \end{pmatrix}$
b. $\begin{pmatrix} 3 & 3 & -2 \\ 4 & 4 & 0 \\ -5 & 0 & 1 \end{pmatrix}$
c. $\begin{pmatrix} 8 & 1 & -1 \\ -4 & 2 & -2 \\ -3 & 0 & -2 \end{pmatrix}$
d. $\begin{pmatrix} 2 & -1 & 1 \\ 4 & 1 & -2 \\ 3 & -1 & 5 \end{pmatrix}$

2.1.6. Rounding to 4 decimal digits, $P^2 = \begin{pmatrix} .9852 & .0247 \\ .0148 & .9753 \end{pmatrix}$, $P^3 = \begin{pmatrix} .9779 & .0368 \\ .0221 & .9632 \end{pmatrix}$,
 $P^{500} = \begin{pmatrix} .6250 & .6250 \\ .3750 & .3750 \end{pmatrix}$. The matrices are the transition matrices for the forest succession model if the time steps were taken to be two years, three years, or five hundred years respectively. Interestingly, the columns of P^{500} are identical and the column entries are in the same ratio as the equilibrium ratio of A trees to B trees that we saw in the text.

2.1.9. a. The transition matrix is $P = \begin{pmatrix} 0 & 0 & 73 \\ .04 & 0 & 0 \\ 0 & .39 & .65 \end{pmatrix}$ with $\mathbf{x}_t = (E_t, L_t, A_t)$.

b. $P^2 = \begin{pmatrix} 0 & 28.47 & 47.45 \\ 0 & 0 & 2.92 \\ .0156 & .2535 & .4225 \end{pmatrix}$, $P^3 = \begin{pmatrix} 1.1388 & 18.5055 & 30.8425 \\ 0 & 1.1388 & 1.898 \\ .01014 & .164775 & 1.413425 \end{pmatrix}$. No-

tice that in P^3 there are now non-zero off-diagonal entries (signifying interaction among the sizes of the classes) and that the (3,3) entry is larger than in the last problem. These are the effects of 65% of the adults living on to the next cycle and reproducing again.

c. All three populations appear to grow roughly exponentially. There is some oscillation in the population values that is particularly noticeable for a small number of iterations. Of course, if 65% of the adults live on into the next time step to produce eggs, the populations should grow even faster than in the previous problem.

Chapter 2.2 1,3,5,6,8,10,12

2.2.1. The matrix for the first insect model is a Leslie matrix, and the matrix for the more complicated insect model is an Usher matrix, where the addition of .65 in the (3,3) position is for the 65% of the adult population that live on into the next reproductive cycle. See problems 2.1.8(a) and 2.1.9(a) for the matrices.

2.2.3. Letting A , B , and C be the matrices in the order given, $\det A = -1$, $A^{-1} = \begin{pmatrix} -3 & 2 \\ 2 & -1 \end{pmatrix}$; $\det B = 8$, $B^{-1} = \begin{pmatrix} 3/8 & 1/8 \\ -1/4 & 1/4 \end{pmatrix}$; $\det C = 0$, so C has no inverse.

2.2.5. a. 3

b. 50%

c. 20% of the organisms in the immature class remain in the immature class with each time step.

d. 30% of the organisms in the immature class progress into the adult class with each time step.

2.2.6. a. $P^{-1} = \begin{pmatrix} -.625 & 3.75 \\ .375 & -.25 \end{pmatrix}$

b. $\mathbf{x}_0 = (1000, 300)$, $\mathbf{x}_2 = (1570, 555)$,

2.2.8. .11 represents the percentage of pups that remain pups after one year. (Pups can not give birth.) One possible explanation for some pups living but not progressing into the yearling stage after one year is that coyotes are born over several months throughout the year. The .15 entries indicate that on average each yearling and adult gives birth to .15 pups each year. The percentage of pups that progress into the yearling stage is 30% each year, so $1 - .11 - .30 = 59\%$ of pups die. While 60% of the yearlings progress into the adult stage, the remaining 40% die. Finally, each year 40% of the adult coyotes die, but 60% live on into the next time step.

2.2.10. a. $(AB)^{-1} = B^{-1}A^{-1} = \begin{pmatrix} -7 & 9 \\ 4 & -5 \end{pmatrix}$, $A^{-1}B^{-1} = \begin{pmatrix} -8 & 3 \\ 11 & -4 \end{pmatrix}$.

b. Answers may vary.

c. Answers may vary.

2.2.12. a. $A_{t+1} = 2/3A_t + 1/4B_t$, $B_{t+1} = 1/3A_t + 3/4B_t$

b. $P = \begin{pmatrix} 2/3 & 1/4 \\ 1/3 & 3/4 \end{pmatrix}$, with $\mathbf{x}_t = (A_t, B_t)$.

c. $P^2 = \begin{pmatrix} 19/36 & 17/48 \\ 17/36 & 31/48 \end{pmatrix}$ so using decimal approximations $A_{t+1} = .5278A_t + .3542B_t$, $B_{t+1} = .4722A_t + .6458B_t$.

d. $P^{-1} = \begin{pmatrix} 9/5 & -3/5 \\ -4/5 & 8/5 \end{pmatrix}$ so $A_{t-1} = 1.8A_t - .6B_t$, $B_{t-1} = -.8A_t + 1.6B_t$.

e. The values of the populations are given in the table below. The populations seem to be stabilizing with $A_t \approx 85.7$ and $B_t \approx 114.3$.

t	0	1	2	3	4	5
A_t	100.0000	91.6667	88.1944	86.7477	86.1449	85.8937
B_t	100.0000	108.3333	111.8056	113.2523	113.8551	114.1063
t	6	7	8	9	10	
A_t	85.7890	85.7454	85.7273	85.7197	85.7165	
B_t	114.2110	114.2546	114.2727	114.2803	114.2835	

e. If the initial populations A_0 and B_0 are non-negative and sum to 200, then they tend toward an equilibrium of around (85.7, 114.3).

2.3.1. The model does behave as expected, showing slow exponential growth in both classes, with decaying oscillations superposed.

2.3.4. a. In zeroing out the first row, no new ungerminated seeds are added to the population. Since the (2, 1) entry has been replaced with zero, no ungerminated seeds progress into the class of sexually immature plants. This eliminates the class of ungerminated seeds from the population. (One reason for considering this model would be to understand the effect of ungerminated seeds on the population dynamics, by imagining what would happen in their absence.)

b. The dominant eigenvalues of the model in the text is 1.1694 and the dominant eigenvalue of the altered matrix is 1.1336. This means that both models predict exponential growth, though the growth rate for the model with no ungerminated seeds is slightly slower. If the ungerminated seed entry of the dominant eigenvectors is discarded, there is also little difference in the stable stage vector for the two models.

c. The ungerminated seeds might be gathered by animals and spread throughout a region, possibly germinating in a later year and spreading the plant species. Also, if the plants have a bad year (due to factors not included in the model, such as drought, extreme cold, fire, etc.) and many fail to survive, the ungerminated seeds still remain in the area despite the temporary adverse growing conditions. If they then germinate at a later date, this may help the population recover. Even though they have little effect on the 'normal year' population dynamics, the ungerminated seeds may well be important.

- 2.3.5. a. The model should produce slow exponential growth. One way to see this is to notice that after one time step 40% of the first class survives to reproduce and 30% remain in the first class. Of the 30%, the model indicates that 40%, or $(.3)(.4) = 12\%$ will survive to reach the reproduction stage after a second time step. This means that at least $.4 + .12 = 52\%$ of the first class will survive to reproduce. Since on average, each adult produces two offspring, we should expect at least $(.52)^2 = 1.04 > 1$ offspring produced by individual members of the first class on average. Thus, the population will grow slowly. In fact, the growth rate should be a little larger than 1.04, since $(.3)^2(.4) = .036 = 3.6\%$ of the first class progress into the second stage after three time steps and then reproduce. Similarly, for four, five, ... time steps. Clearly, the situation is somewhat complicated and an eigenvalue analysis can help us understand the growth trend more easily.
- b. The eigenvalue 1.0569 is dominant with eigenvector $(.9353, .3540)$. The other eigenvalue is $-.7569$ with corresponding eigenvector $(-.8841, .4672)$.
- c. The intrinsic growth rate is 1.0569, a number a little bit bigger than 1.04 as anticipated by (a). The stable stage distribution is $(2.6423, 1)$.
- d. Using eigenvectors calculated by MATLAB, $(5, 5) = 9.0100(.9353, .3540) + 3.8757(-.8841, .4672)$.
- e. $\mathbf{x}_t = 9.0100(1.0569)^t(.9353, .3540) + 3.8757(-.7569)^t(-.8841, .4672)$.
- 2.3.7. The dominant eigenvalue is .6791 so the coyote population will decline rather rapidly. The stable stage distribution is $(2.2636, 1, 7.5877)$.
- 2.3.9. a. The transition matrix $P = \begin{pmatrix} 0 & 5 \\ 1/6 & 1/4 \end{pmatrix}$ is for an Usher model.
- b. The dominant eigenvalue is 1.0464 with eigenvector $(.9788, .2048)$. The other eigenvalue is $-.7964$ with eigenvector $(-.9876, .1573)$.
- c. The intrinsic growth rate is 1.0464 and the population will grow. The stable stage distribution is $(4.7783, 1)$.
- 2.4.1. a. $A: \lambda_1 = 1$ and $\lambda_2 = .6$; $B: \lambda_1 = -1$ and $\lambda_2 = 5$; $C: \lambda_1 = -3$ and $\lambda_2 = 2$
- b. $A: \mathbf{v}_1 = (3, 1), \mathbf{v}_2 = (1, -1)$; $B: \mathbf{v}_1 = (-2, 1), \mathbf{v}_2 = (1, 1)$; $C: \mathbf{v}_1 = (-3, 2), \mathbf{v}_2 = (1, 1)$