1. In a finite population of size N there are two types of agent A and B. If A has fitness r and B has fitness 1, then the fixation probability for A is

$$\rho_A = \frac{1 - 1/r}{1 - 1/r^N} \quad \text{if} \quad r \neq 1$$

and

$$\rho_A = \frac{1}{N} \quad \text{if} \quad r = 1$$

   (a) For the case when $r = 2$ evaluate ρ_A for $N = 2, 3, 4$ and $N \to \infty$. What trend do you notice?

   (b) Show that ρ_A is a decreasing function of N, for all values of r. Explain why. What is the minimum value of ρ_A for general r.

   (c) Prove that

$$\rho_A > \frac{1}{N}$$

   when $r > 1$. (Hint: You will probably find it useful to look up the formula for the sum of a geometric series.)

2. Each of the following games is played in a population of size N, with weak selection.

	A	B
A	10	0
B	5	2

	A	B
A	1	8
B	2	10

For each game:

(a) Consider the case where N is large. Determine whether A or B are selected in favor of fixating?

(b) Consider the case where $N = 2$. Are either A or B selected in favor of fixating?

(c) For which values of N is A an ESS. For which values of N is B an ESS?

(d) Now take these games as deterministic rather than stochastic. What are the pure ESS's. How do these answers compare to your answers in (c)?

3. Consider the repeated prisoner's dilemma game in a finite population of size N. In each turn a player chooses to cooperate by giving the opponent a benefit b while incurring a cost c to itself ($b > c$), or defect by giving no benefit and incurring no cost. The game is repeated m times. Suppose the population only contains Tit-4-Tat and All-D strategists.

   (a) Write down the game matrix for this repeated game.

   (b) If N is large, what is the minimum value of m for T4T to be selected in favor of fixating? Express your answer in terms of b and c. What is the minimum value of m for the case where the benefit is twice the cost? (Note: in the deterministic repeated prisoner's dilemma the minimum number value of m is b/c. How do the two results compare? Which is larger and why?)

   (c) If $N = 2$, T4T can never be selected in favor of fixating, why not?

   (d) If $N = 3$, T4T can be selected in favor of of fixating. How large must m be for this to be the case? Express your answer in terms of b and c. What is the minimum value of m for the case where the benefit is twice the cost?

4. Challenge for the mathematically adventurous: For the following game played in a population of size N, with weak selection,

	A	B
A	a	b
B	c	d

show that the fixation probability is given by

$$\rho_A = \frac{1}{N} \frac{1}{1 - \frac{w}{6}(N(a + 2b - c - 2d) + 4d - 2a - b - c)}$$

You may find the following results helpful. If $w << 1$

$$\frac{1}{1-w} \approx 1 + w, \qquad \prod_{i=1}^{n}(1 + \alpha_i \omega) \approx 1 + \sum_{i=1}^{n} \alpha_i \omega$$

and

$$\sum_{i=1}^{n} i = \frac{1}{2}n(n+1) \quad \text{and} \quad \sum_{i=1}^{n} i^2 = \frac{1}{6}n(n+1)(2n+1)$$