1. Birds form winter flocks of two members that stay together through the winter. Individuals may choose to watch for predators, or not watch. If at least one of the pair watches. both members survive the winter. If a bird is a member of a flock in which neither watches, there is a 50% chance it will be killed. Non-watchers get more time to eat and so if they survive the winter they have 5 offspring. Watchers only have 4 offspring.

   (a) Write down the evolutionary game matrix for this interaction
   When two non watchers meet, half the time they get 5 and half the time they die (which is - 1 point – the population has been reduced by 1).

	W	N
W	4	4
N	5	$\frac{1}{2}(5) - \frac{1}{2}(1) = 2$

   (b) Are either Watching or Non-Watching strategies ESS?
   Neither are ESS. since for Watchers $4 < 5$ and for non Watchers $2 < 4$.

   (c) What will be the mixed strategy ESS? The mixed strategy is when they have equal fitness. Let x be the probability of watching and $1 - x$ the probability of non watching then the payoffs to a watcher is $4x + 4(1 - x) = 4$ and the payoff to a non-watcher is $5x + 2(1 - x) = 3x + 2$. The equilibrium comes when $4 = 3x + 2$ or $x = \frac{2}{3}$. So the birds should watch $\frac{2}{3}$ of the time and not watch $\frac{1}{3}$ of the time. Note they could do better if they had some way of taking turns watching.

2. Consider the Hawk-Dove game with a resource worth 10 fitness points and cost of 20 points.

   (a) Write down the payoff matrix and find the equilibrium distribution of Hawks and Doves.

	H	D
H	$\frac{1}{2}(10 - 20) = -5$	10
D	0	$\frac{1}{2}10 = 5$

   Let x be the proportion of Hawks and $1-x$ the proportion of Doves. Then the fitness of Hawks is $f_H = -5x+10(1-x) = 10-15x$ and the fitness of doves is $f_D = (1-x)5 = 5-5x$. Setting $f_H = f_D$ we get

$$10 - 15x = 5 - 5x \;\Rightarrow\; 10x = 5 \;\Rightarrow\; x = \frac{1}{2}$$

   . So the population is 50% Doves and 50% Hawks.

   (b) Now suppose that when two Dove's meet they each incur a small cost of 2 fitness points each because they waste time strutting around without actually fighting. What is the new payoff matrix and the new equilibrium distribution.

	H	D
H	-5	10
D	0	3

Let x be the proportion of Hawks and $1-x$ the proportion of Doves. Then the fitness of Hawks is $f_H = -5x+10(1-x) = 10-15x$ and the fitness of doves is $f_D = (1-x)3 = 3-3x$. Setting $f_H = f_D$ we get

$$10 - 15x = 3 - 3x \;\Rightarrow\; 12x = 7 \;\Rightarrow\; x = \tfrac{7}{12}$$

. So the population is $\frac{5}{12}$ are Doves and $\frac{7}{12}$ are Hawks.

(c) Now lets add the Bully strategy. Bully is a strategy that threatens, but runs away if the opponent also threatens (ie a Bully acts like a Dove against a Hawk and like a Hawk against a Dove) Assume that when two Bully's meet, the first to threaten makes the other run away. From this interaction half the time a Bully will get 0 and half the time it will get 10 so that the average payoff is 5. Two Bullies meeting incur no cost because they don't waste time strutting around like Dove do. Write down the 3×3 matrix for this interaction. Show that Bullies are fitter than Doves so that Dove die out. Find the equilibrium distribution of Bullies and Hawks.

	H	D	B
H	-5	10	10
D	0	3	0
B	0	10	5

Against Doves Bullies get 10 and Doves get 3. Against Bullies get 5 and Doves 0. Against Hawks they each get nothing. So Doves are less fit than Bullies and so die out. When Doves do die out Hawks and Bullies come to a 50/50 equilibrium as in part (a).

(d) Now consider a fourth strategy called Retaliator. A Retaliator struts around like a Dove unless threatened, in which case it fights like a Hawk. Bullies run away from Retaliators and Hawks fight them. A Retaliator behaves like a Dove with Doves and other Retaliators. Write down the full 4×4 matrix. Show that Retaliators are an ESS.

	H	D	B	R
H	-5	10	10	-5
D	0	3	0	3
B	0	10	5	0
R	-5	3	10	3

Retaliators are a weak ESS because Retaliators do better against Retaliators than Hawks and Bullies do. Doves do as well against Retaliators as Retaliators do, and they both do equally well against Doves.

(e) In a population of only Doves and Retaliators both are equally fit. However, if there are too many Doves the population is susceptible to invasion by Bullies. Consider a population of x Doves and $1-x$ Retaliators. Find the fitness of an invading Bully in this population and show that it is fitter than Doves if the portion of Doves is too high. For what values of x is the population safe from invasions by Bullies.
The fitness of Bullies in this population is $f_B = x10$. The fitness of a Dove is 3, so Bullies are less fit than Doves when $10x < 3$ or $x < \frac{10}{3}$.

(f) For what values of x is the population of Doves and Retaliator's safe from invasions by Hawks?
The fitness of Hawks is $f_H = 10x - 5(1-x) = 15x - 5$. Hawks are less fit than Doves when $15x - 5 < 3 \Rightarrow x < \frac{8}{15}$.

(g) This question illustrates an important point about interactions involving competition for a scare resource. The population as a whole is most fit when no fighting takes place, but unless individuals are willing to fight if threatened, the population ends up being dominated by Bullies and Hawks. However, it turns out that a population of only Retaliators is not the most fit population we could imagine, since they waste time strutting around when they meet. Suppose it happens that during each interaction, one player is assigned the label Hawk and the other Dove, then on average the resource is shared without any fighting or strutting. Compare the fitness of such a population to that of one of only Retaliators. What signals might animals use to decide who gets to be Hawk and who gets to be Dove when they meet? Discuss some examples in nature.
In the Hawk meets Dove interaction the total payoff is 10 for an average payoff over time of 5 per individual. In the case of a population of retaliators the payoff is 3. In nature we could have animals decided who gets the benefit by marking territories, and if the benefit is in your territory you get to keep it.

3. Consider the following payoff matrix for three strategies A, B, C with relative frequency x, y, and z respectively.

$$\begin{pmatrix} 0.2 & 0.0 & 0.3 \\ 0.4 & 0.1 & 0.0 \\ 0.1 & 0.2 & 0.1 \end{pmatrix}$$

(a) Explain why there are no pure ESS's.
A is not an ESS because $0.2 < 0.4$. B is not an ESS because $0.1 < 0.2$. C is not an ESS because $0.1 < 0.3$.

(b) Find the equilibrium distribution of the strategies.
The equilibrium comes when they are all equally fit. The first equality is $f_a = f_b$:

$$0.2x + 0.3z = 0.4x + 0.1y \Rightarrow 2x + 3(1-x-y) = 4x + y \Rightarrow 5x + 4y = 3$$

The second equality is $f_b = f_c$:

$$0.4x + 0.1y = 0.1x + 0.2y + 0.1z \Rightarrow 4x + y = x + 2y + (1-x-y) \Rightarrow 4x = 1 \Rightarrow x = \tfrac{1}{4}.$$

Inserting this into the first equation gives:

$$5(\tfrac{1}{4}) + 4y = 3 \Rightarrow 5 + 16y = 12 \Rightarrow y = \tfrac{7}{16}\ .$$

Which gives $z = 1 - \frac{4}{16} - \frac{7}{16} = \frac{5}{16}$.

(c) Show that the matrix can be transformed to the form

$$\begin{pmatrix} 0 & -a_2 & b_3 \\ b_1 & 0 & -a_3 \\ -a_1 & b_2 & 0 \end{pmatrix}$$

and specify the values of the a's and b's indicating that there is a cycling between the three strategies around the equilibrium.

Subtracting the diagonal element in each column we get

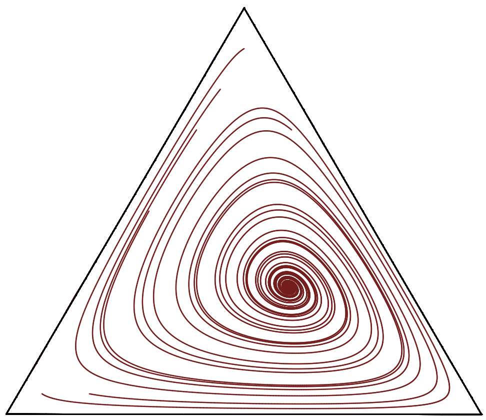
$$\begin{pmatrix} 0 & -0.1 & 0.2 \\ 0.2 & 0 & -0.1 \\ -0.1 & 0.1 & 0 \end{pmatrix}$$

.

(d) Hence determine the stability of the equilibrium and the direction of the circulation.

$a_1a_2a_3 = (0.1)(0.1)(0.1) = 0.001$ and $b_1b_2b_3 = (0.2)(0.1)(0.2) = 0.004$. The determinant is $b_1b_2b_3 - a_1a_2a_3 = 0.004 - 0.001 = 0.003$. This is positive, so the equilibrium is stable.

(e) Sketch the phase simplex for this system.



(f) Use the replicator equations in Excel to sketch a phase portrait for y vs x.