# Homework Week 3 Two Populations

**Part I**

1. Joe surmises that cute endangered animals elicit more donations from the general public than ugly animals. To test this, he sends postcards showing an adorable, cuddly, baby ape to 100 individuals randomly selected from the local phone book and an envelope soliciting donations. He sends postcards depicting an older ape with some hair loss and a scar across its forehead to another 100 randomly selected individuals with the same donation request envelope. He then counts the donations received in response to each of the two postcards.

A. Is this a test of two independent samples or a matched pairs test?

Independent samples

B. What are Ho and Ha?

$H_0: \mu_{CUTE} - \mu_{UGLY} = 0 \quad H_a: \mu_c - \mu_u > 0$

C. Assuming an underlying normal distributions, for x bar (cute) = \$7500 and s (cute) = \$100, x bar (ugly) = \$6000 and s (ugly) = \$500, what is the value of the associated test statistic? We don't know sigma, so we need a t statistic.

Are variances equal? No, since $s_{CUTE} \leq 4 * s_{UGLY}$.

For unequal variances $t = \dfrac{(\bar{x}_c - \bar{x}_u) - (\mu_c - \mu_u)}{\sqrt{(s_c^2/n_c) + (s_u^2/n_u)}}$

$t = 1500/\sqrt{100 + 2500} = 1500/50.99 = 29.42$

2. Tammy has been doing field work in Africa and wants to examine the impact of exposure to the sun on the vitamin content of the cornmeal used in cooking. She measures the vitamin content of 25 bowls newly processed cornmeal. She then places that cornmeal on a sunny window ledge and measures the vitamin content after 10 days.

A. Assuming normality, would this be a test of two independent samples or a matched pairs test? Matched pairs

B. What are Ho and Ha? $H_0: \mu_D = 0 \quad H_a: \mu_D \neq 0$

C. Write the equation (using symbols!) for the appropriate test statistic.

$t_D = \dfrac{\bar{X}_D - \mu_D}{s_D/\sqrt{n_D}}$ with $df = n_D - 1$ and $D$ = difference

3. Mitch wants to examine the affect of a new water filtration system on the levels of petroleum products in the water that runs off streets during and after a storm. Should he use two independent samples or matched pairs to study this? Why?

Matched pairs. He will want to sample the water stream before ~~and~~ entering and after exiting the filtration system. The difference in petroleum levels would reflect how much the system removed from the passing water.

4. *Eileen has found some great data on a U.S. government site that shows the snowfall* (in inches) at various locations in the Cascades in 1980 and in 2010. The data take the form

| 1980 | $Rank_{80}$ | 2010 | $Rank_{10}$ |
|---|---|---|---|
| 56 inches | 13 | 34 inches | 2 |
| ~~45~~ | 5 | ~~66~~ | 15 |
| 39 | 4 | 73 | 17 |
| ~~52~~ | 12 | ~~45~~ | 6 |
| ~~66~~ | 16 | 37 | 3 |
| 57 | 14 | ~~52~~ | 11 |
| ~~49~~ | 8 | 48 | 7 |
| 22 | 1 | 50 | 10 |
| 74 | 19 | ~~49~~ | 9 |
| | $T_{80} = 91$ | | $T_{10} = 80$ |

Assuming the underlying populations are normal, should Eileen analyze these data using a test for independent samples or a matched pairs test? Why?

Independent samples. There is no link between the measurements in 1980 and those in 2010.

Eileen finds out the underlying populations are extremely non-normal. Which test should she use? Wilcoxon Rank Sum Test

What is the value of the test statistic for that test?

$$T = T_{80} = 91 \quad \text{(as Independent Samples)}$$

$$E(T) = \frac{n_{80}(n_{80} + n_{10} + 1)}{2} = \frac{9(9+9+1)}{2} = 85.5$$

$$\sigma(T) = \sqrt{\frac{n_{80} n_{10}(n_{80} + n_{10} + 1)}{12}} = \sqrt{\frac{1539}{12}} = 11.325$$

$$z = \frac{T - E(T)}{\sigma_T} = 0.4856$$

As matched pairs:

| 1980 | 2010 | Diff. | $R^+$ | $R^-$ |
|---|---|---|---|---|
| 56 | 34 | 22 | 5 | |
| 45 | 66 | -21 | | 4 |
| 39 | 73 | -34 | | 9 |
| 52 | 45 | 7 | 3 | |
| 66 | 37 | 29 | 8 | |
| 57 | 52 | 5 | 2 | |
| 49 | 48 | 1 | 1 | |
| 22 | 50 | -28 | | 7 |
| 74 | 49 | 25 | 6 | |
| | | | $T_+ = 25$ | $T_- = 20$ |

$$E(T) = \frac{n(n+1)}{4} = \frac{9(10)}{4} = 22.5$$

$$\sigma(T) = \sqrt{\frac{n(n+1)(2n+1)}{24}} = \sqrt{\frac{9(10)(19)}{24}} = 8.44$$

$$z = \frac{T_+ - E(T)}{\sigma(T)} = \frac{25 - 22.5}{8.44} = 0.296$$