Circles, Angles, and Vector Components - Workshop

This workshop is an exploration of angles and trigonometric functions using graphical and analytical methods. The last part of the workshop is an application to the problem of finding vector components V_x and V_y given a vector V in direction θ.

Materials: This workshop write-up, a Trig Table handout, graph paper, a handout with a 10 cm centered circle on graph paper, colored pencils.

Part 1 - Calculating X and Y vector components given vector magnitude and direction
In this first part of the workshop we'll set up a table of vectors and their components so that in the last part of the workshop we can fill in magnitudes using the analytical techniques of Ch 3.3 of the physics text.

1. Using your protractor and ruler, draw a vector V on your graph paper with a magnitude of 10 cm (one decimeter (dm)) and a direction of 30 degrees (as measured counterclockwise from the positive x axis). Draw the V_x and V_y component vectors of V in the head-to-tail layout.
    a. Measure and label your vector diagram with the magnitude of the V_x and V_y component vectors.
    b. Use the Pythagorean theorem to verify that your measured component vectors produce the magnitude of V (you can use 1dm in place of 10cm to scale the numbers by 10 if you choose - but be consistent throughout the workshop).
    c. In an appropriate class notebook copy the following table and fill out the V_x and V_y columns of the table for your 30 degree angle.

θ (deg)	V_x		V_y	
30				
45				
60				
55				
0				
90				

2. Now fill out V_x and V_y columns in the rest of the table for a 1 dm (10 cm) vector directed according to the various angles in column one. This may take awhile because you'll need to measure the V_x and V_y components of the vector at each of the indicated angles. Don't worry about the third and fifth columns for now.

Part 2 - Trigonometric tables
In this middle part of the workshop we'll develop the analytical techniques of trigonometry so we can apply the techniques to finding vector components. In particular, we'll start building a table of trigonometric functions that allow us calculate lengths when we're given angles. The precalculus reading for this week covers these topics in depth. The purpose of the workshop is to engage in active reading and learning using the textbook ideas and techniques.

1. The magnitude of the V_x component of your vector is determined by the cosine (cos) of the angle and the V_y component of your vector is determined by the sine (sin) of the angle. In this part of the workshop we'll explore the sine and cosine functions (y = sin(θ), y = cos(θ)) as they are viewed on a circle. We'll pick a few points on the circle, draw right triangles, and build a sine and cosine function table for a some angles using the Pythagorean theorem.
    a. In your notebook, build the following empty sine and cosine table.

Angle θ	cos(θ)	sin(θ)

    b. Pull out your graph paper handout with the 10 cm circle. We'll be using this circle to fill in the trig table above. You should see a coordinate grid with a circle centered at (0,0) with radius 10 cm.
    c. Read Section 5.3 p321-322 of your precalculus text and produce a diagram for Example 1 on p 322, but double the radius and adjust the coordinates of the points accordingly. Your diagram should have the point (x,y) = (6,8) on the circumference of the circle, and a triangle as shown in the figure at the bottom of p321 of your precalculus textbook. For the purpose of this lab, let's call this triangle the *circle trig triangle* (CTT). (Mathematicians insist on naming everything and computer scientists make acronyms out of everything.)
    d. Use the Pythagorean theorem to verify the correctness of the two sides and the hypotenuse of the triangle for Example 1 p322 of the precalculus text.
    e. Use your protractor to measure the angle θ. Fill in the sine and cosine values for the angle in your trig table. Note that the the value for the cosine of the angle is the x coordinate divided by the circle radius and the value for the sine of the angle is the y coordinate divided by the radius, as explained in the precalculus text.
3. The previous step of the lab constructed an angle from a convenient point, but the angle

itself was not a convenient angle. In this step we will try to discover the sine and cosine values for more convenient angles.

   a. On the circumference of your 10cm circle construct the point (x,y) with x = 5. Construct the CTT.
   b. At this point you know the cos(θ) value, so fill it in the next row of your angle table. Measure the angle θ and fill it in the table on the same row.
   c. Use the pythagorean theorem and some algebra to calculate the sin(θ) value and put that in the table. Leave the value as an exact value with the square root - don't use your calculator to obtain a numerical value yet. Leave room next to the exact value to put a numerical value later.

4. Repeat the previous step of the lab for a point (x,y) on your circle with y = 5. By the end of this step you should have three angles in your table with sine and cosine values filled in.
5. Repeat the previous step of the lab for the two circle points (x,y) = (0,10) and (x,y) = (10,0). Note that you will not actually be able to build the CTT, but it should be obvious without the CTT what the angles are and what the corresponding sine and cosine values are for the angles. This should give you two more entries in your table. If you're confused about this, then look at Example 2 p322 in your precalculus text.
6. There's one more convenient angle that would be nice to have in the table - a 45 degree angle. Use your protractor to draw a 45 degree angle on your 10 cm circle and then make the CTT. Use the Pythagorean theorem to calculate the sine and cosine values of the 45 degree angle. Again, leave the values in square root form rather than converting them to numbers (with room for the number later). Fill in your table for the 45 degree angle.
7. Now is a convenient time to calculate your square root expressions into actual numbers and add them to your trigonometric tables, so do that (put both the expression and the number in your table entry with an equal sign (=) between them).
8. Use the Trigonometric Tables handout to verify the cos(θ) and sin(θ) values for each angle in column one of your trig table.

Part 3 - Vector Components

In this final part of the workshop we return to fill out the vector table of part 1 using analytical techniques of trigonometry developed in the second part.

1. The magnitude of the V_x component of your vector is determined by the cosine (cos) of the angle and the V_y component of your vector is determined by the sine (sin) of the angle.
   a. Return to your earlier vector table and label the third column with cos(θ) and label the fifth column with sin(θ).
   b. Use the trigonometric table that you built in the previous step of this lab to fill in the cos(θ) and sin(θ) values for each row according to the angle in column one. Note that there is one angle that we don't have in our Trig table, so you can use the handout table for that angle.
   c. What is the mathematical relationship between the values in the V_x column and the values in the cos(θ) column?

d. What is the mathematical relationship between the values in the V_y column and the values in the sin(θ) column?

2. If you have time remaining, start another table identical to the table above and prepare to fill it out for a vector R with magnitude 20 cm = 2 dm. If you don't have time, skip to the next step.
   a. Fill out the V_x and V_y columns of this new table by measuring angles as you did for the table of a 10 cm (1 dm) vector above.
   b. This time use your calculator to determine the cos(θ) and sin(θ) values of each of the angles. You'll need to set your calculator to degrees or use the Trig Table handout to translate your calculator's answer in radians to degrees. Radians are just another scale to measure angles (like miles vs kilometers) that your calculator likes better.
   c. Verify that the values you get are the same as the values you obtained from the Trig Table handout.
   d. Compare the values in the cos(θ) and sin(θ) columns with the values in the V_x and V_y columns (respectively) of this new 2 dm vector table. What is the mathematical relationship between V_x and cos(θ)? What is the mathematical relationship between V_y and sin(θ)?

3. Postulate a general formula for the V_x and V_y components of a vector V of direction θ and magnitude k in terms of the cos(θ) and the sin(θ) (respectively).
   a. Check your formula for a vector of 1.5 dm at the angles in the tables.
   b. Consult your physics Ch 3.3 reading to verify your formulas for the magnitude of the V_x and V_y vector components of the vector in each row.

You now know how to calculate the magnitude of the V_x and V_y components of any vector.