Patterning Math Lab 4a

This lab is an exploration of transformations of functions, a topic covered in your Precalculus textbook in Section 1.5. As you do the exercises in this lab you will be closely reading the text and examples and reconstructing them in Desmos.

Launch Desmos and bring up a copy of the Precalculus textbook Ch 1.5 (you can find this through your WAMAP account or by finding the link to the textbook through the program web-site).

Vertical Shift

a) Read the short section in the center of p. 62 of the textbook on the vertical shift.
b) Enter the equation of the "Try it Now" exercise on p. 63. Scale your window so the graph is conveniently visible in the window (if you don't know how to do this, search the Knowledge Base for Windows settings). Find a formula for a function b(t) in terms of h(t) as described in the "Try it Now" exercise. Enter this formula into Desmos. While you're at it, replace the constant 10 in exercise with a slider parameter (say, p).
c) What are the coordinates of the point representing the launch of the ball following the function h(t)? What are the coordinates of the point representing the launch of the ball following the function b(t)? Can you determine those two launch points just from the b(t) and h(t) formulas? Explain.
d) Transform the h(t) formula into *vertex form*. You may need to review your notes on Ch 3.2.
e) Use your vertex form formula for h(t) to determine the coordinates of the highest point reached by the ball following the function h(t). Check your answer using the graph.
f) Can you create the vertex form of b(t) from the vertex form of h(t) easily? Explain how. Verify your answer by determining the vertex of the function b(t) and checking your coordinate numbers on the graph.

Horizontal Shift

a) Read Example 3 p. 63 in the textbook and then read the short section near the top of p. 64 on the horizontal shift.
b) In Desmos, enter the function f(x) = x^2. On the next line, enter the expression f(x+2). Describe the transformation of the function f(x) into the function f(x+2) as you see it in Desmos.
c) Change the expression f(x+2) into the expression f(x+a) and make a into a slider parameter. Slide the a parameter back and forth and explain what happens. Why does a positive value of the parameter a make the graph shift to the left instead of the right? Explain this observation. Read Example 5 on p. 65 of the text to help with your explanation. Also read the very important comment in the paragraph right above Example 6 on p. 65 of the text. What does it mean to apply a shift to the "input" vs the "output" of a function? Give your answer using examples from your experimental work above.

Combining Shift Transformations

a) Enter into Desmos the function g(x) = |x|, the absolute value function.
b) Now create a new function h(x) in Desmos constructed from the function g(x) using a horizontal shift left by 1 and a vertical shift down by 3. You might do this by first creating the horizontal shifted version and then enhancing it with the additional vertical shift. Verify your results by reading Example 7 p. 66. When you combine shift transformations, does it matter what order you apply them? Explain.

Vertical Stretch or Compression

a) Read the paragraph at the bottom of p. 72 of the textbook on Vertical Stretch/Compression.
b) Redisplay your function f(x) = x^2 in Desmos (or re-enter it if you have lost it). Create a new function g(x) = k * f(x), where the star (*) is just simple multiplication, and let k be a slider. Set the k slider to k = 1 and verify a few points on the graph to see that they are what you expect. For example, you should see the points (-5,25) and (5,25) on the graph of g(x). Check a few other common squares (like 4^2, 3^2). The f and g functions should be the same.
c) Now set the k slider to k = 2. Determine the g(x) values for the values x = 3, x = 4, x = 5. What pattern do you see? Is this a stretch or a compression of the vertical values?
d) Set the k slider to k=½. What should the g(x) values for x = 3, x = 4, x = 5 be now? Is this a vertical stretch or compression? Verify the results on your graph.
e) What happens when the k slider goes negative?

Horizontal Stretch or Compression

a) Read the paragraph at the bottom of p. 74 of the textbook on Horizontal Stretch/Compression. Carefully notice that the stretch/compression factor (the k) is applied on the *input* to the function whereas the vertical stretch/compression factor was applied to the *output* of the function.
b) Redisplay your function f(x) = x^2. Create a new function h(x) = f(k*x). (Undisplay your g(x) vertical stretch/compression function for now). Set the k slider to k = 1 and verify that the f and h functions are the same.
c) Set the k slider to k = 2. Explain the pattern you see. Is this a stretch or a compression? What is being stretched or compressed?
d) Set the k slider to k = ½. Explain the pattern you see. Is this a stretch or a compression? What is being stretched or compressed?

Combining Vertical Transformations - Shift and Stretch/Compression

a) What happens if we do both a Shift and a Stretch/Compression transformation together? First start with just vertical combinations.
b) Read the paragraph in the middle of p. 76 on Combining transformations along with the short rule for combining vertical transformations.
c) Modify your g(x) = k*f(x) vertical stretch/compression transformation above so it also has a vertical shift. Play with both the stretch/compression and the shift parameters to see that the combination works as you expect.

Combining Horizontal Transformations - Shift and Stretch/Compression

a) This section is extra, but adds to the symmetry of the explorations. You might enjoy exploring the interaction of the transformations discussed in this lab.
b) Read the top of p. 77 in the textbook on combining horizontal transformations.
c) Modify your h(x) = f(k*x) horizontal stretch/compression transformation above so it also has a horizontal shift. Play with both the stretch/compression and the shift parameters to see that the combination works as you expect. What happens when the stretch/compression parameter switches from positive to negative and negative to positive? Can you explain what you see?
d) What does the text mean when it says that it is much easier to horizontally stretch before shifting? Try hand calculating the value of a sample stretch/compression after a shift and before a shift. Is one way really harder than the other?

Patterning Math/Physics Lab 4b

There are 3 main parts. Part I continues Physics Lab 5, and has you continue and extend the analysis and discussion of the data from that lab. Part II has you use video analysis to explore motion in 2 dimensions. Part III gives you some experience with vectors and some tools for representing them pictorially and switching between the magnitude & direction and perpendicular component forms as well as for adding vectors that supplements your ruler/protractor constructions.

Part I: Analysis/Discussion of Physics Lab 5

This continues and expands the Analysis section from Physics Lab 5.

- If you were present for this lab, sit with your lab partners at adjacent computers. Make sure to do your own LoggerPro analysis (zooming on graphs, fitting lines and curves, etc.), but discuss the questions with your partners, and compare the results from your analysis.
- If you were not present for this lab, work with someone else who also was not present for the lab. Consult with a classmate who was present, and request their permission to use her/his group's data. You must acknowledge the source of your data (include the names of all the members of the lab group whose data you are using). Make sure to do your own LoggerPro analysis (zooming on graphs, fitting lines and curves, etc.), but discuss the questions with your partners, and compare the results from your analysis.

a) Examine the position vs. time and the velocity vs. time graphs for the level track data. See if you can identify the three 'parts' of the trip: 1) when the cart is moving freely away from the motion detector (after being pushed), 2) while the spring is being compressed and expanding during the rebound phase, and 3) when the cart is moving freely towards the motion detector. It might be easier to distinguish the three parts in one of the graphs as compared to the other (which graph?). In addition, see if you can identify the turn-around point on both graphs. Discuss how the two graphs are related in the 3 parts of the cart's trip.

b) The manufacturer claims that when the cart rolls freely on a level track, the friction is very low and the cart moves with nearly constant velocity. If this were true, what would the position vs. time graph look like when the cart is moving freely away from the motion detector and when the cart is moving freely towards the motion detector? Do the position vs. time and velocity vs. time graphs support or refute the manufacturer's claim? Explain your reasoning.

c) Change the graphs to show individual data points: right click on each graph, select Graph Options, in the Graph Options tab, turn on Point Symbols and turn off Connect Points. Why do this?

d) Fit a line to the portion of the position vs. time data where it moves freely away from the motion detector. Avoid the 'bounce' part of the trip. The velocity vs. time graph should help you pick the appropriate time window to fit the line for the position vs. time graph (why?). Record the slope (and units) from the best fit line. What physical quantity does the slope of a position vs. time graph tell you? What does the sign of your result mean?

e) For that same time window, on the velocity vs. time graph, determine the mean (average) velocity using the Statistics tool (either under Analyze: Statistics or you can find the Statistics button on the toolbar near the Linear Fit button). Record the mean value for the velocity (did you remember to include units?). Compare with your result from part d).

f) Save your graphs, copy/print/cut/paste/label into your notebook, etc.

g) Examine the position vs. time and the velocity vs. time graphs for one of the angled track data (your choice). See if you can identify the three parts of the trip (these parts repeat): 1) when the cart is moving freely down the ramp, 2) while the spring is being compressed and expanding during the rebound phase, and 3) when the cart is moving freely up the ramp, and repeat. In this motion, there are two different kinds of turn-around points: what are they, and what do they correspond to in the cart's motion graphs?

h) Look again at the position vs. time and the velocity vs. time graphs for the angled track data of your choice. Does this pattern look familiar – have you seen position vs. time and velocity vs. time graphs that have a similar shape?

i) As before, change both graphs to show individual data points. Zoom in on the first full up and down part of the trip (first bounce to second bounce) for which you have good data. As you did with the bouncing ball data (hope I didn't give away the answer to part d). Oops – guess I just did), highlight the data in a region centered around the turn-around point at the top of the track and away from the bounce events. Fit a line to the velocity vs. time data, and record the slope (did you remember the units?). Also, fit a quadratic to the position vs. time data for the same time window, and record the value for A (what should the units for A be?). Save your graphs, copy/print/cut/paste/label into your notebook, etc.
j) Repeat for the other angles (graphs in notebook), and organize your results into a table, with columns: angle, slope, A value (and leave room for more columns). (Since the analysis here is similar for the different angles, you don't need to include all these graphs)
k) Based on your data/analysis, is it reasonable to assume that (at a particular angle) the cart moves with constant acceleration as it moves up and down the ramp (not including the bounce event)? Explain your reasoning.
l) For constant acceleration, the slope of a linear fit to a velocity vs. time graph and the A value for a quadratic fit to a position vs. time graph are each related to the acceleration. How? Add two new columns to your table: acceleration from the velocity vs. time analysis and acceleration from the position vs. time analysis. Are these consistent for a given angle?

Part III: Two Dimensional Motion

We have seen that using the Motion Detector is a powerful tool for measuring position vs. time (and calculating velocity vs. time) for one dimensional motion. The nature of this measuring device means that it isn't good for measuring two dimensional motion. For that, video analysis is better suited. If you completed Math/Physics Lab 2b (from week 2), you should have good experience with video analysis in one dimension – the extension to two dimensions is straightforward. Here you will work with a partially analyzed video. If you are unsure how you would start with a movie file and obtain the data as shown, you need to go back to Math/Physics Lab 2b and complete it on your own time.

a) Open LoggerPro. Under File: Open, find the Sample Movies folder. In that folder, find the Basketball Shot folder. Open Basketball shot with analysis (**not** Basketball shot vector analysis).
b) Right click on the movie, and choose Movie Options. Change the speed to 0.25 x Original, click OK, and play the movie. Play it slower if you'd like. Watch several times.
c) Examine the graph of X position and Y position vs. time. When (at what time) and where (X position and Y position) does the ball bounce off the ground? How do you know (what features on the graph did you use)?
d) Make a graph of X velocity and Y velocity vs. time. Use Insert: Graph; the new graph might already have Y velocity vs. time, so just add in X velocity (right click on the graph, choose Graph Options, under the Axes Option tab, scroll down the Y-Axis Columns and select X velocity. Resize and move the graphs so that the velocity vs. time graph is directly under the position vs. time graph and so that the horizontal axes match up (like the default setting with the motion detector).
e) Look at the X position vs. time graph (the red dots) and the X velocity vs. time graph (these might be red triangles) while the ball is in the air before it bounces. What does this tell you about the motion in the x direction, e.g. no motion, constant (non-zero) velocity, constant (non-zero) acceleration, other, etc.? What specifically do you use to make this conclusion?
f) Hopefully you concluded that the ball moved with constant velocity in the x direction. Determine the value of this constant velocity using what should hopefully by now be familiar methods. Record your results.
g) Look at the Y position vs. time graph (the blue dots) and the Y velocity vs. time graph (these might be blue diamonds – does this sound like a Lucky Charms commercial?) while the ball is in the air before it bounces. What does this tell you about the motion in the y direction, e.g. no motion, constant (non-zero) velocity, constant (non-zero) acceleration, other, etc.? What specifically do you use to make this conclusion?

h) Hopefully you concluded that the ball moved with constant acceleration in the y direction. Determine the value of this constant acceleration using what should hopefully by now be familiar methods. Record your results. What remarkable numerical value do you obtain? Does this make sense?
i) Compare the motion diagram (the dots on the movie) with the Y position vs. time graph. They look superficially similar. Comment on the similarities and differences in appearances of the two.
j) Hopefully you noted that while the general shapes were similar, they didn't match exactly in the y-direction or along the horizontal axis. The y-direction can be taken care of by changing the vertical scale on the position vs. time graph (try it and see). Why won't changing the scaling on the horizontal axis also have a similar matching effect?
k) Instead, make a Y position vs. X position graph. Insert a new graph as before. Click on the vertical axis label name and choose Y. Click on the horizontal axis label name and choose X. Resize and position this graph as needed to compare it to the motion diagram on the movie. Compare these two.
l) Save this file with your analysis to your cubbie.
m) Now, open Basketball Shot vector analysis. Set the movie so that it will play at 0.1 x Original, then watch it. Watch several times as needed.
n) The red arrows represent the velocity vector of the basketball. The grey arrows are the x and y components of the velocity vector.
o) What do you notice about the magnitude of the x component of the velocity vectors? Does this make sense?
p) What do you notice about the magnitude and direction of the x component of the velocity vectors? Does this make sense?
q) What do you notice about the magnitude and direction of the y component of the velocity vectors? Does this make sense?
r) If you'd like to learn to make Animated Vector Displays like in this file, in LoggerPro, File: Open, then navigate your way to Experiments: Sample Data: Physics: Animated Display Vectors: Exploring Animated Displays.

Part II: Vectors – representations and addition

a) The interactive Vector Addition program demonstrated in Physics Lecture #3 is available at http://phet.colorado.edu/en/simulation/vector-addition (click on the green Run Now! button).
b) Pull an arrow from the Grab one bucket, and put it in the middle of the screen. Figure out how to move the vector around without changing its magnitude or direction. Figure out how to change its magnitude and direction without changing the location of its tail (the non arrow-head end; the arrow-head end is the tip of the vector). Play around with the different Show Components styles. Click the Show Grid box.
c) Add a second vector to the screen. Place the two vectors 'tip to tail': the arrow end of one vector touches the other end of the second vector. Turn on Show Sum, and move the new vector to form a triangle with the other two. Basically play around with vectors and vector addition to get a better sense of the concepts from the reading.
d) Return to just one vector on the screen. The first problem in this week's Physics Problem Set has a vector with $R_x = –22.2$ units and $R_y = 27.8$ units. This program can only have integer units for the vector components (as programmed, the vector must fit cleanly on the coordinate grid). However, we can round for a close approximation to Problem 1. Adjust the vector so that = –22 and $R_y = 27$. What are the magnitude and direction of this vector? Compare this to the answer for Problem 1; you should be very close.
e) Confirm this magnitude using the Pythagorean theorem. Make sure to draw the vector, the corresponding right triangle, label the legs of the triangle (the components) with the right numbers, and show your calculation from the Pythagorean theorem.

f) Use the program to approximate Problem 2. We can't get 187 km and 35^o north of east, but we can get 18.9, which is close with a scaling factor of 10, and 32^o north of east. Try it, and compare the results from the program to the answer to Problem 2 (don't forget the scaling factor); this time it won't be as close as for Problem 1.

g) Use the program to approximate Problem 3. Get as close to 18.9 and 35^o north of west as you can. How should you represent 35^o north of west as an angle measured counterclockwise from the +x axis? Recall that due west corresponds to 180^o. Compare to the answer to Problem 3; it again won't be as close as Problem 1.

h) While this program is a fun tool for getting a sense of vector representations (magnitude & direction vs. components) and for learning about vector addition, we see that the limitations of the coordinate grid make it less useful for arbitrary vectors. Instead, go to http://www.mathsisfun.com/algebra/vector-calculator.html. Here, you can enter any magnitude & direction or pair of components. If you scroll down, it will show you the graphical representation as well. Use this tool to try Problem 2 again, and compare to the answer. Try Problem 3 again.

i) This tool also will add up to four vectors, and show the graphical representation. Use this tool to investigate Problem 4 on the physics problem set. Make sure to use the Pythagorean theorem to confirm that the magnitude of the resultant vector is consistent with the components of the resultant vector (use the same steps as before).